

## Unit 6.4- Transformations of Trigonometric Functions

### Parent Sinusoidal Functions:

- The parent sinusoidal functions that we have learned thus far are \_\_\_\_\_
- Parent functions are the \_\_\_\_\_ form of the function. Just like any other parent function, we can apply \_\_\_\_\_ to them.

### Transformations of Sinusoidal Functions:

Let  $y = \sin x$  and  $y = \cos x$  be the parent functions. There are 5 transformations that can be applied to the parent function.

$$y = -a \sin [k(x - d)] + c$$

$$y = -a \cos [k(x - d)] + c$$

-		-appear outside of function : affects y-values
a		
c		
k		-appear inside of function : affects x-values
d		

As can be seen, these variables [a, c, k, d] cause the same transformations that you have seen for the past two years. However, in the context of trigonometric functions, these variables have slightly different interpretations.

### Graphing Trigonometric Functions

In order to graph trigonometric functions, we will fill out the following chart each time:

Equation of Axis	_____	Period	_____
Amplitude	_____	Zeros/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

### Features of the Chart:

- Column to the left: factors that only affect the \_\_\_\_ variable [will be used to create the y-axis]
- Column to the right: factors that only affect the \_\_\_\_ variable [will be used to create the x-axis]

### Terminology of the Chart:

- Equation of Axis*: determined by the \_\_\_\_\_ value [ $y=c$ ]
- Amplitude*: determined by the \_\_\_\_\_ value
- Max Value*: can be determined by the formula: \_\_\_\_\_
- Min Value*: can be determined by the formula: \_\_\_\_\_
- Period*: can be determined by the formula: \_\_\_\_\_

- *Zeroes/Max/Min*: indicate how often a new key point appears. It is determined by the formula: \_\_\_\_\_.
- *Phase Shift*: a synonym for horizontal \_\_\_\_\_. It is determined by \_\_\_\_\_.
- *Template*: indicates the \_\_\_\_\_ on the x-axis (what value will you count by)

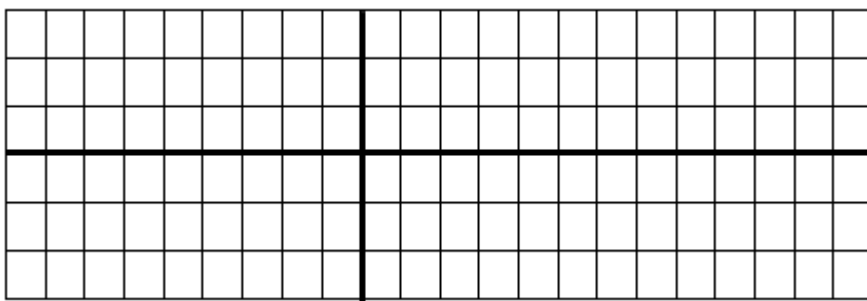
In summary,

$$y = -a \sin [k(x - d)] + c$$

### Parent Functions

*Exercise:* Graph  $f(x)=\sin x$  and  $f(x)=\cos x$  using a table of values. Then, fill out the chart for each parent function.

**y=sinx**



Pattern for y=sinx:

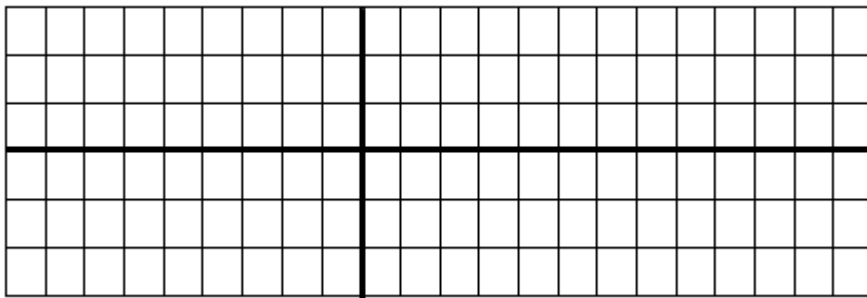
\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
Sin or Cos _____			
Reflection? _____			

**y=cosx**



Pattern for y=cosx:

\_\_\_\_\_

\_\_\_\_\_

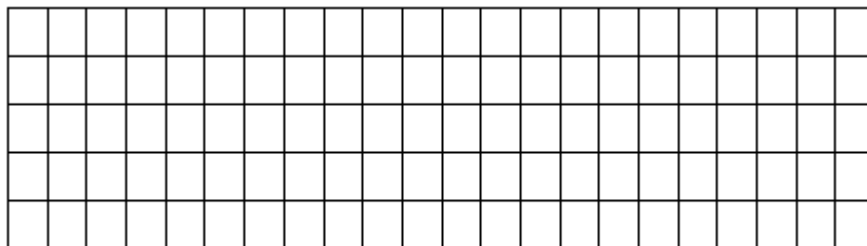
\_\_\_\_\_

Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
Sin or Cos _____			
Reflection? _____			

**Transformations 1 and 2: Amplitude and Equation of Axis [The effect of 'a' and 'c']****Amplitude and Equation of Axis:**

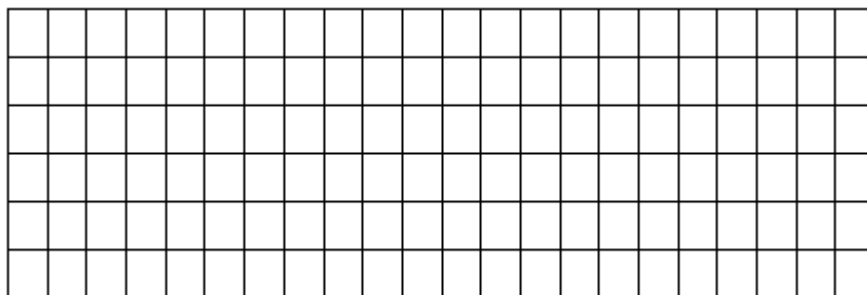
- affect the \_\_\_\_\_ and \_\_\_\_\_ values of the sinusoidal function

*Example:* Graph  $y=2\sin x$ .



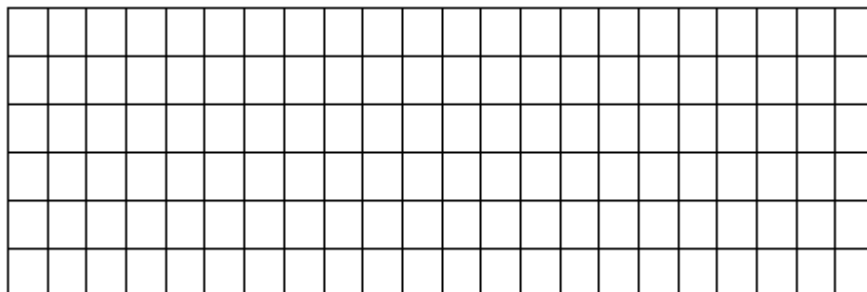
Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

*Example:* Graph  $y=3\cos x$ .



Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

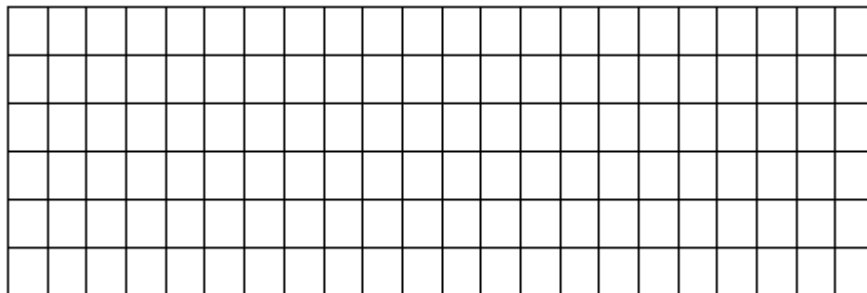
*Example:* Graph  $y=2\cos x + 3$ .



Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____

Sin or Cos \_\_\_\_\_  
 Reflection? \_\_\_\_\_

Example: Graph  $y = \frac{1}{2} \cos x - 4$ .



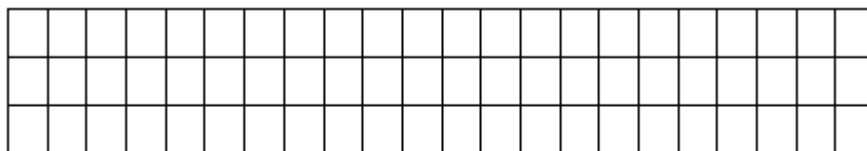
Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

### **Transformation 3: Reflection across the x-axis [The effect of '-']**

#### **Reflection across the x-axis:**

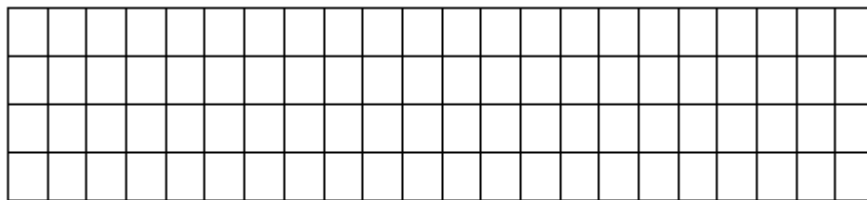
- affects the pattern for  $y = \sin x$  and  $y = \cos x$
- Normally, the pattern for  $y = \sin x$  is \_\_\_\_\_.  
This becomes \_\_\_\_\_.
- Normally, the pattern for  $y = \cos x$  is \_\_\_\_\_.  
This becomes \_\_\_\_\_.

Example: Graph  $y = -\cos x$ .



Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

Example: Graph  $y = -2\sin x + 2$ .



Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____

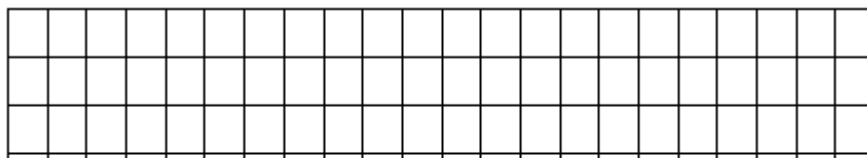
Min Value _____	Sin or Cos _____ Reflection? _____	Template _____
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### **Transformation 4: Period [The effect of 'k']**

#### **Period:**

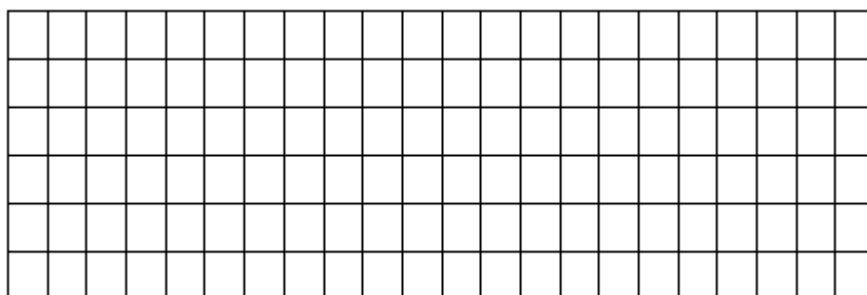
- Period is determined by the formula: \_\_\_\_\_
- The period affects the zeroes/max/min value because \_\_\_\_\_.  
Therefore, the period affects how often every new key point appears.

*Example:* Graph  $y = \sin 3x$ .



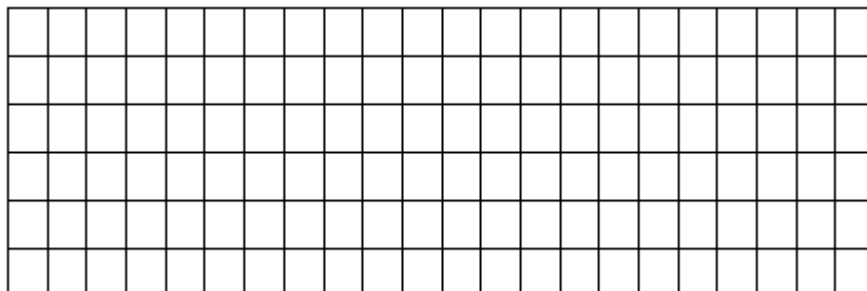
Equation of Axis _____	Sin or Cos _____ Reflection? _____	Period _____
Amplitude _____		Zeroes/Max/Min _____
Max Value _____		Phase Shift _____
Min Value _____		Template _____

*Example:* Graph  $y = 3\sin \frac{1}{2}x + 1$ .



Equation of Axis _____	Sin or Cos _____ Reflection? _____	Period _____
Amplitude _____		Zeroes/Max/Min _____
Max Value _____		Phase Shift _____
Min Value _____		Template _____

*Example:* Graph  $y = -10\cos 3x - 2$ .



Equation of Axis _____	Sin or Cos _____ Reflection? _____	Period _____
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Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

### Transformation 5: Phase Shift [The effect of 'd']

Homework: Handout C7-1-3

#### Phase Shift:

- This is equivalent to a horizontal shift \_\_\_\_ to the left or right. Remember that you move in the \_\_\_\_\_ direction as expected.
  - Positive d-value: move \_\_\_\_\_
  - Negative d-value: move \_\_\_\_\_
- The phase shift can affect the template. The template will be the \_\_\_\_\_ between the zeroes/max/min value and the phase shift value.

Example: Graph  $y = \cos(x - \frac{\pi}{3})$


Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

Example: Graph  $y = \sin(x + \frac{\pi}{4})$


Equation of Axis	_____	Period	_____
Amplitude	_____	Zeroes/Max/Min	_____
Max Value	_____	Phase Shift	_____
Min Value	_____	Template	_____
	Sin or Cos _____		
	Reflection? _____		

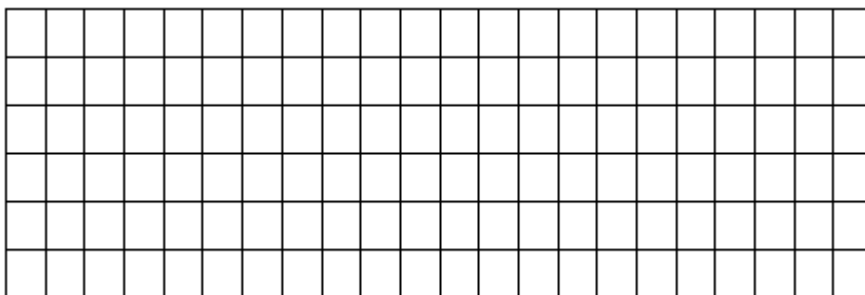
Exercise: What is the period of  $f(x) = \cos(\frac{2}{3}x)$ ?

Exercise: What is the period, amplitude and phase shift of  $f(x) = -5\sin(3x - \frac{\pi}{4})$ ?

**Note:** There should be nothing attached to the x in the bracket. If there is, it must be factored.

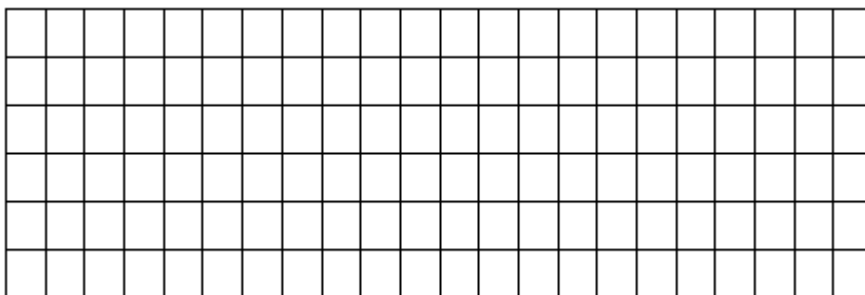
**Combining all of the Transformations**

*Example:* Graph  $y = 2\sin[\frac{1}{2}(x - \frac{\pi}{4})] + 1$



Equation of Axis	<u>                    </u>	Period	<u>                    </u>
Amplitude	<u>                    </u>	Zeroes/Max/Min	<u>                    </u>
Max Value	<u>                    </u>	Phase Shift	<u>                    </u>
Min Value	<u>                    </u>	Template	<u>                    </u>
	Sin or Cos <u>                    </u>		
	Reflection? <u>                    </u>		

*Example:* Graph  $y = -2\cos(2x - \frac{2\pi}{3}) + 3$



Equation of Axis	<u>                    </u>	Period	<u>                    </u>
Amplitude	<u>                    </u>	Zeroes/Max/Min	<u>                    </u>
Max Value	<u>                    </u>	Phase Shift	<u>                    </u>
Min Value	<u>                    </u>	Template	<u>                    </u>
	Sin or Cos <u>                    </u>		
	Reflection? <u>                    </u>		

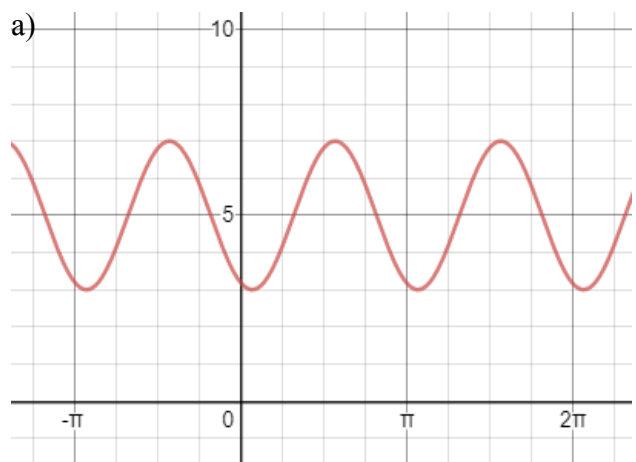
**Homework:** Handout C# 4

## Determining Equations of Trigonometric Functions

Steps:

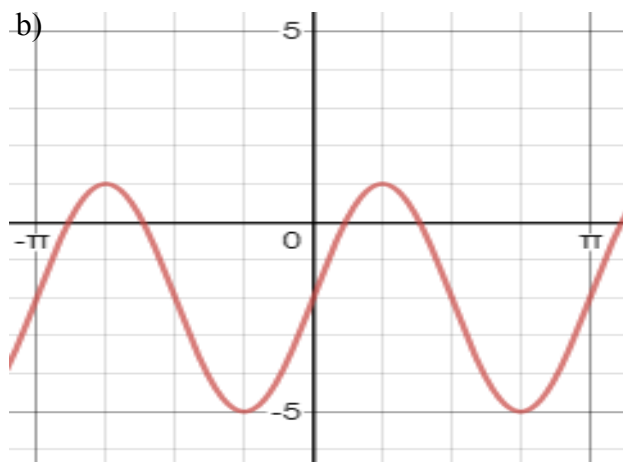
1. Determine whether you want to create a sine function or a cosine function. Use the following templates:  $\_\_\_\_\_\_ \sin [\_\_\_\_\_\_ (\theta \_\_\_\_\_\_)] \_\_\_\_\_\_$  or  $\_\_\_\_\_\_ \cos [\_\_\_\_\_\_ (\theta \_\_\_\_\_\_)] \_\_\_\_\_\_$ .
2. Find the equation of the axis of symmetry and write the “c” value in the template.
3. Find the amplitude and write the “a” value in the template.
4. Find the period and write the appropriate “k” value in the template. Remember that  $\text{period} = 360/k$  so, you need to solve for the appropriate k value.
5. Choose a starting point on the graph:
  - When creating a sine function, the point must be  $\_\_\_\_\_\_$ .
  - When creating a cosine function, the point must be a  $\_\_\_\_\_\_$ .
6. Using the starting point, determine if a “-” is required and whether there is a phase shift. If there is a phase shift, write the “d” value in the template.

*Exercise:* Determine a sine and cosine function for each sinusoidal model.



$$f(\theta) = \_\_\_\_\_\_ \sin [\_\_\_\_\_\_ (\theta \_\_\_\_\_\_)] \_\_\_\_\_\_$$

$$f(\theta) = \_\_\_\_\_\_ \cos [\_\_\_\_\_\_ (\theta \_\_\_\_\_\_)] \_\_\_\_\_\_$$



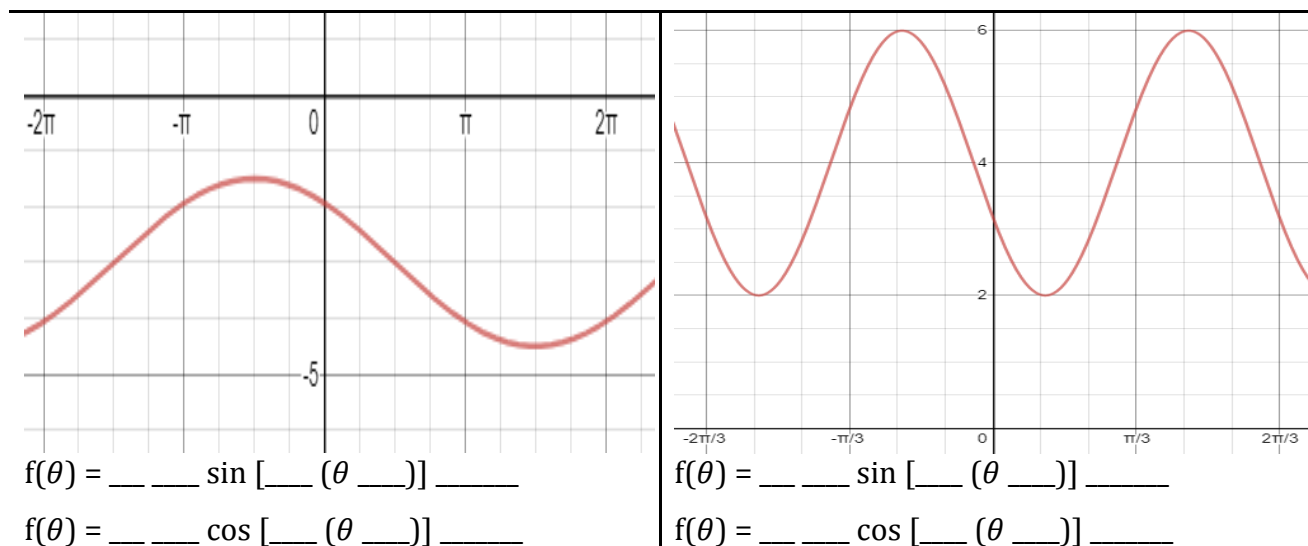
$$f(\theta) = \_\_\_\_\_\_ \sin [\_\_\_\_\_\_ (\theta \_\_\_\_\_\_)] \_\_\_\_\_\_$$

$$f(\theta) = \_\_\_\_\_\_ \cos [\_\_\_\_\_\_ (\theta \_\_\_\_\_\_)] \_\_\_\_\_\_$$

c)

d)





**Homework:** Handout D#1 and Page 1, 4-8, 12, 14

## Handout C- Graphing Transformations of Sine and Cosine

### Instructions:

- Graph each from  $0 \leq x \leq 2\pi$  if period is  $2\pi$  or less
- Graph a minimum of 1 complete cycle if period is greater than  $2\pi$
- Complete the chart for each graph:

Equation of Axis	<u>                    </u>	Period	<u>                    </u>
Amplitude	<u>                    </u>	Zeroes/Max/Min	<u>                    </u>
Max Value	<u>                    </u>	Phase Shift	<u>                    </u>
Min Value	<u>                    </u>	Template	<u>                    </u>
Sin or Cos <u>                    </u>			
Reflection? <u>                    </u>			

The independent axis ( $x$  axis) *must be correctly labelled* in order to facilitate all transformations. Graphs must be neat, legible, and accurate.

1a)  $y = 2 \sin \theta$     b)  $y = \cos \theta + 1$     c)  $y = -\sin \theta$     d)  $y = -2 \cos \theta$     e)  $y = 3 \cos \theta - 4$

2a)  $y = \sin 2\theta$     b)  $y = -2 \cos 3\theta$     c)  $y = 4 \sin \frac{1}{2} \theta$     d)  $y = -2 \sin 2\theta + 3$     e)  $y = -3 \cos 4\theta + 2$

3a)  $y = \sin(\theta + \frac{\pi}{2})$     b)  $y = \frac{1}{2} \cos(\theta + \frac{\pi}{4})$     c)  $y = -4 \sin(\theta + \frac{\pi}{6}) + 2$     d)  $y = 2 \cos(\theta - \frac{\pi}{3}) - 1$

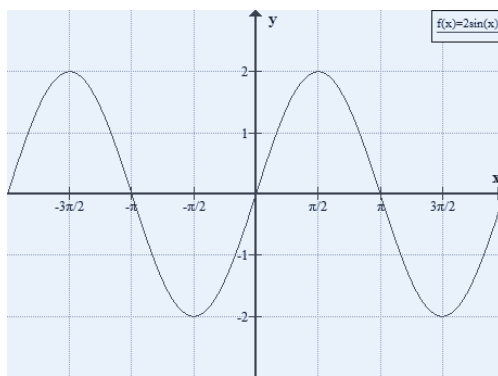
4a)  $y = -\sin 2(\theta - \frac{\pi}{3})$     b)  $y = \sin \frac{1}{2}(x - \frac{\pi}{2}) - 2$     c)  $y = \cos 3(x + \frac{\pi}{3}) + 2$     d)  $y = 3 \cos 2(\theta + \frac{\pi}{3})$

e)  $y = 3 \cos(2x + \frac{\pi}{3})$     f)  $y = \frac{1}{2} \sin(2x - \frac{\pi}{6}) - 3$     g)  $y = -\cos(2x - \frac{\pi}{2})$

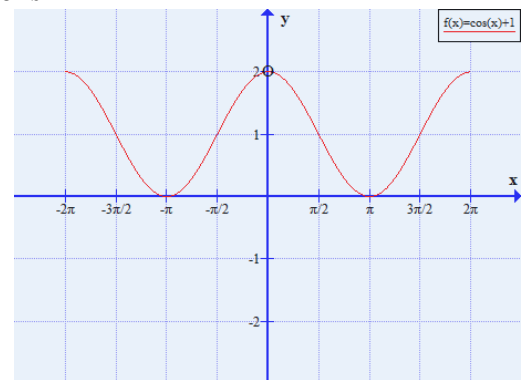
## Solutions Handout C

## Graphing Trig Functions

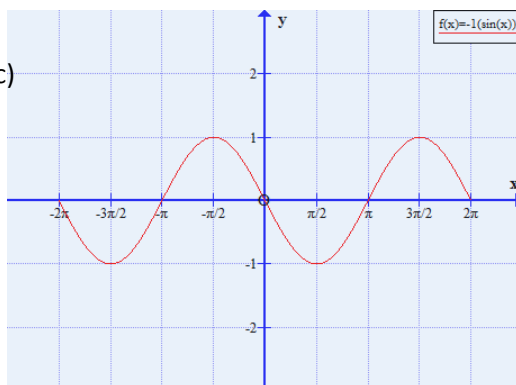
1a)



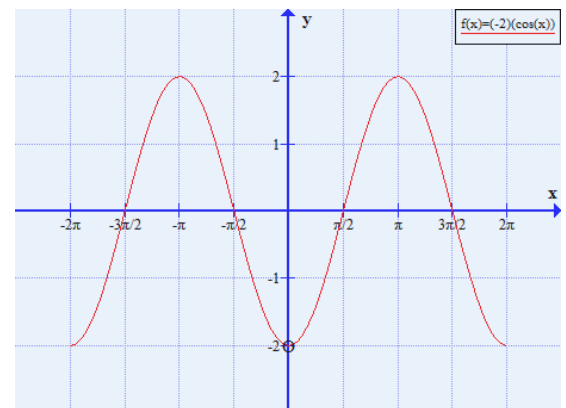
b)



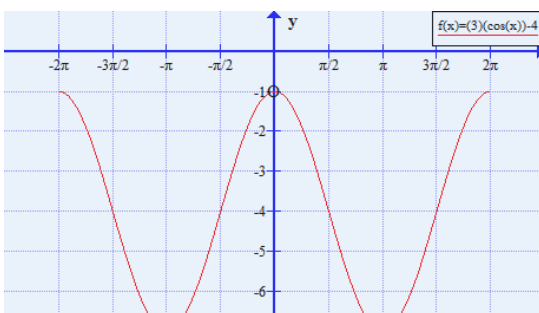
1c)



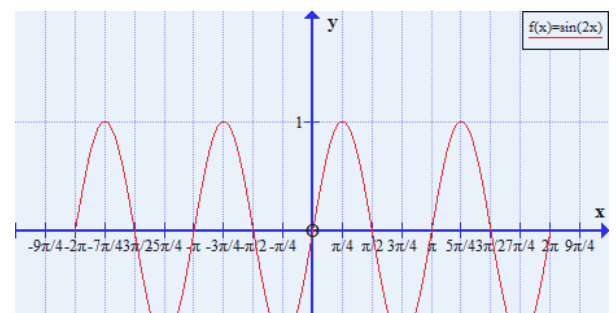
1d)



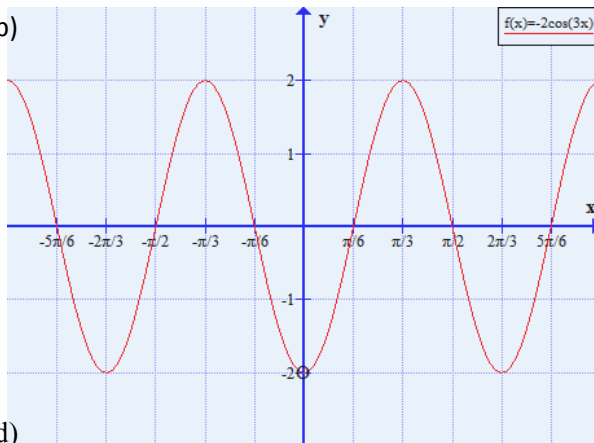
1e)



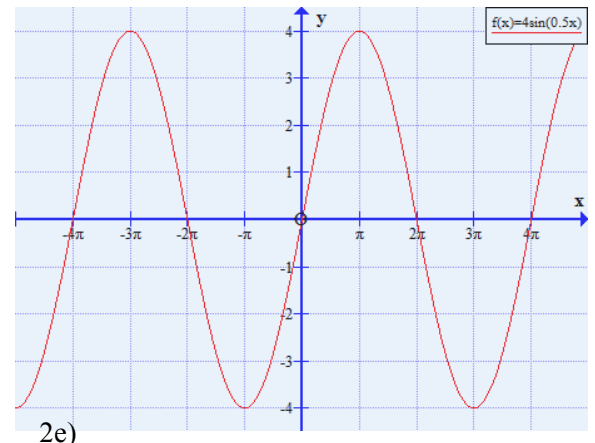
2a)



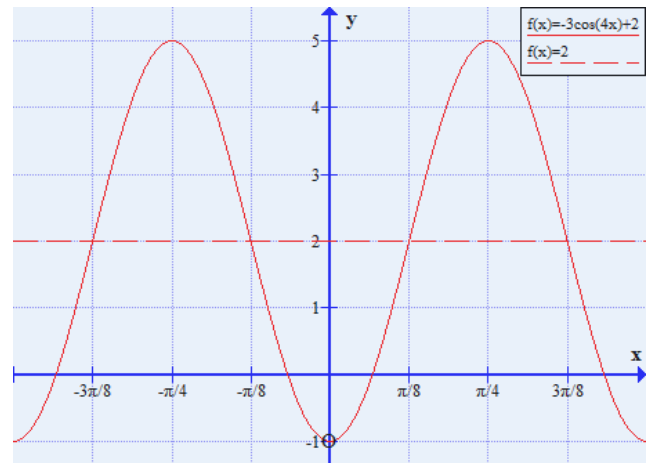
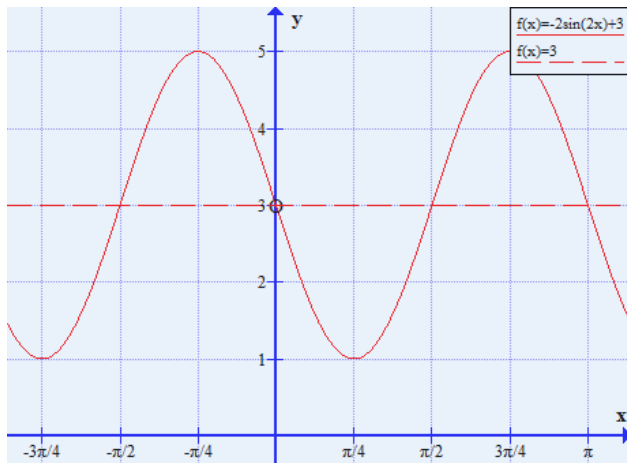
2b)



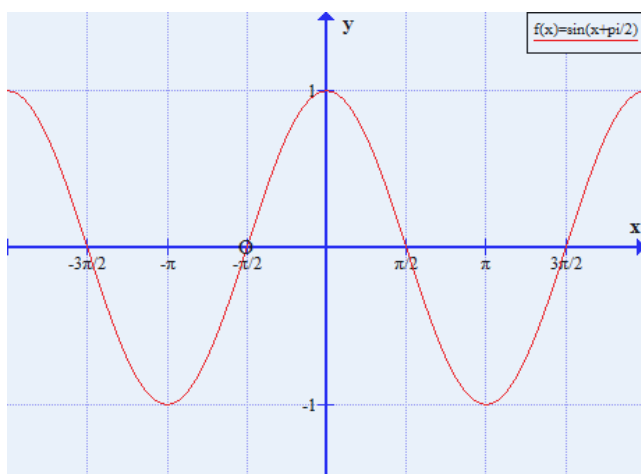
2c)



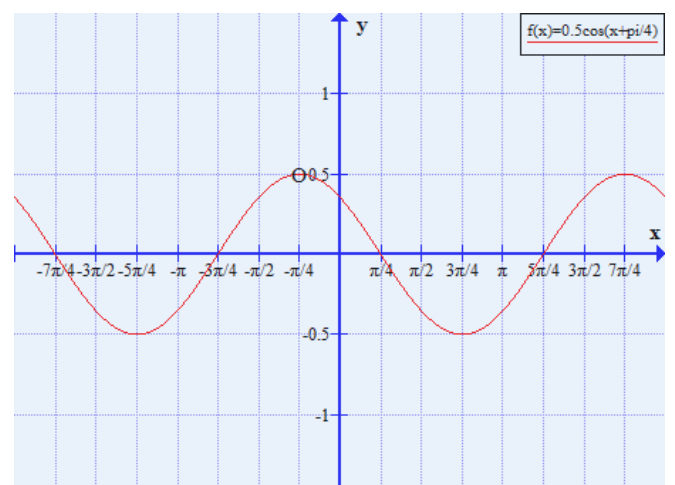
2d)



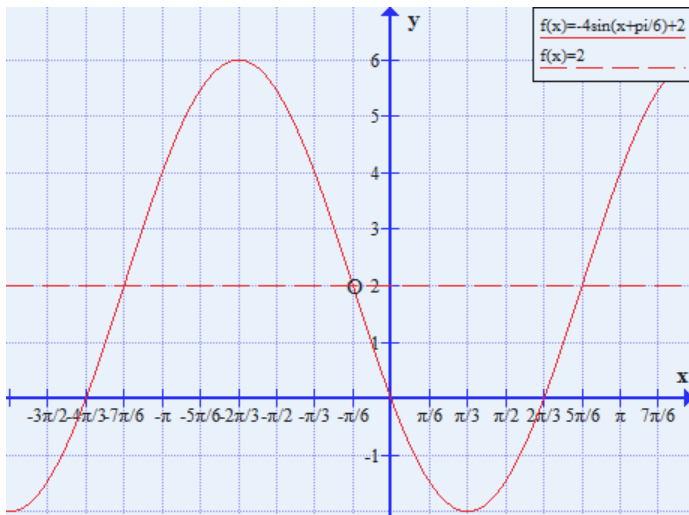
3a)



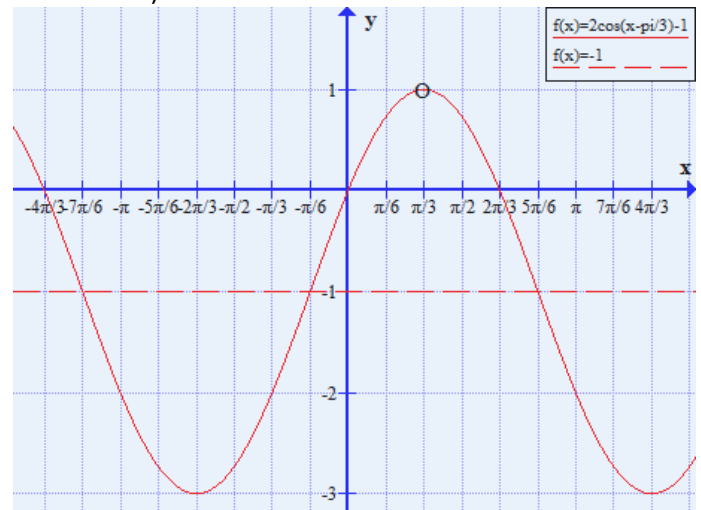
3b)



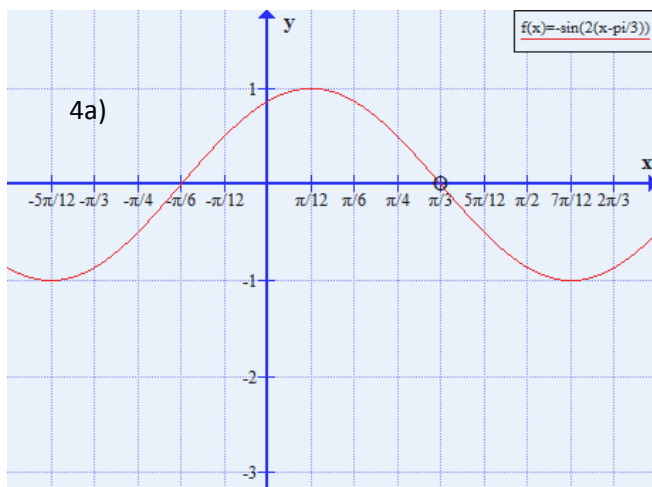
3c)



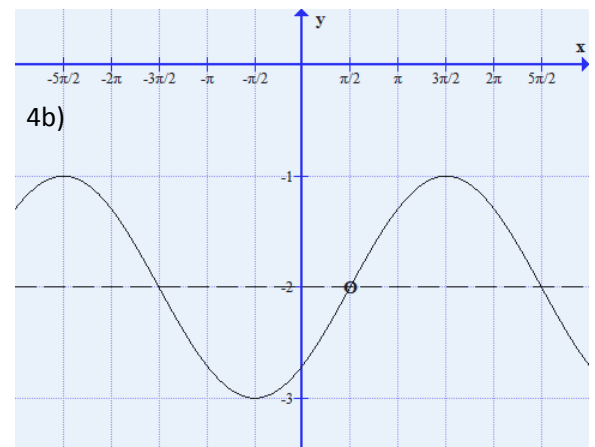
3d)



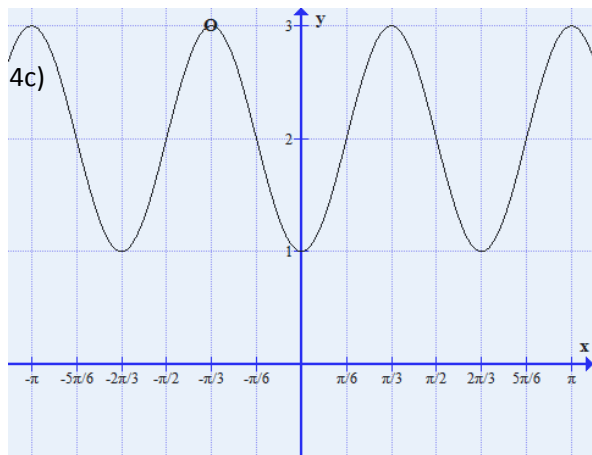
4a)



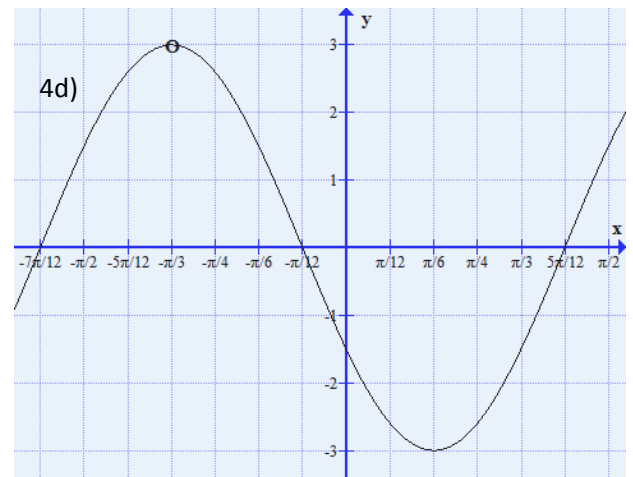
4b)



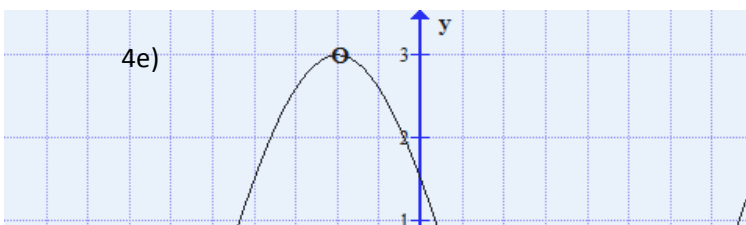
4c)



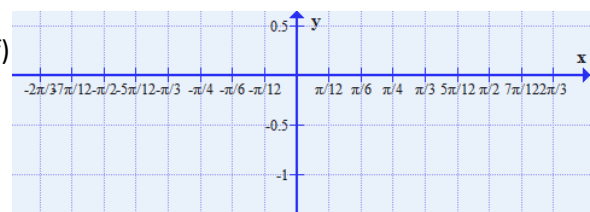
4d)

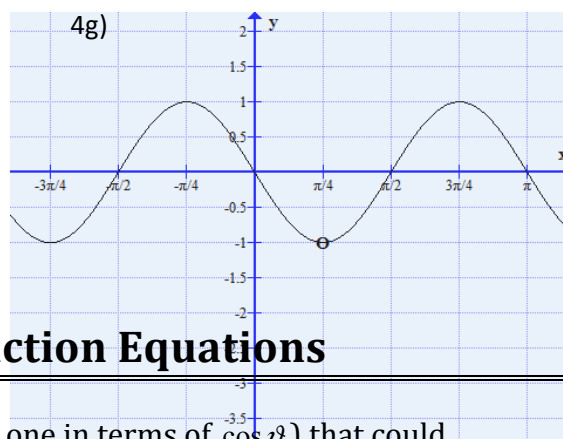


4e)



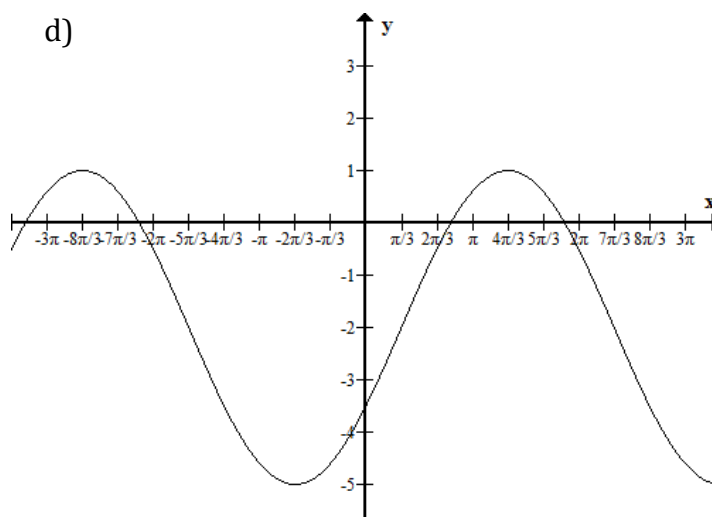
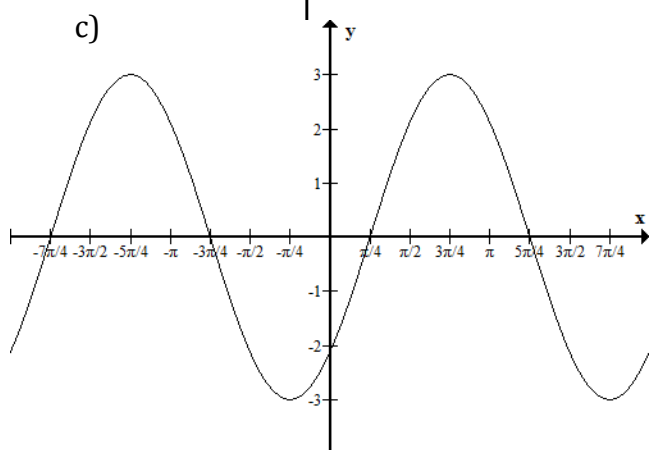
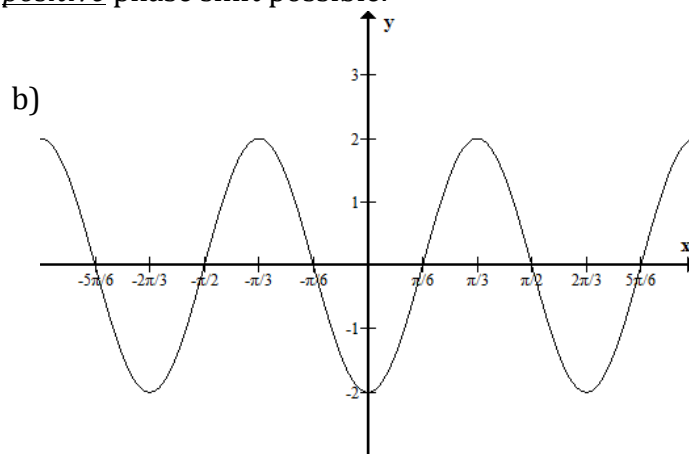
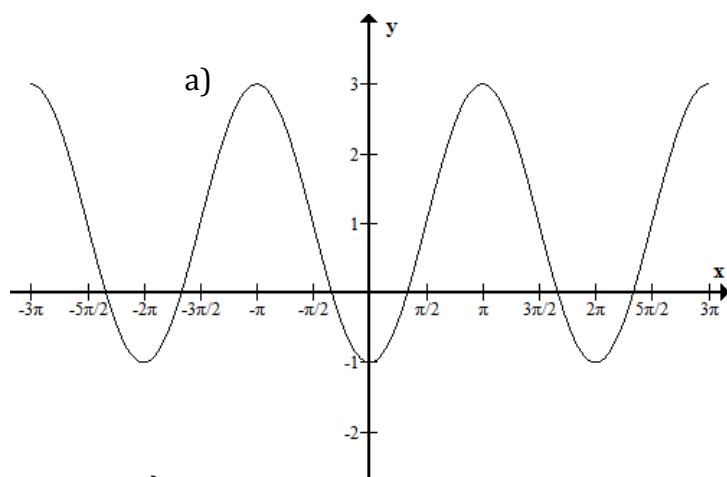
4f)



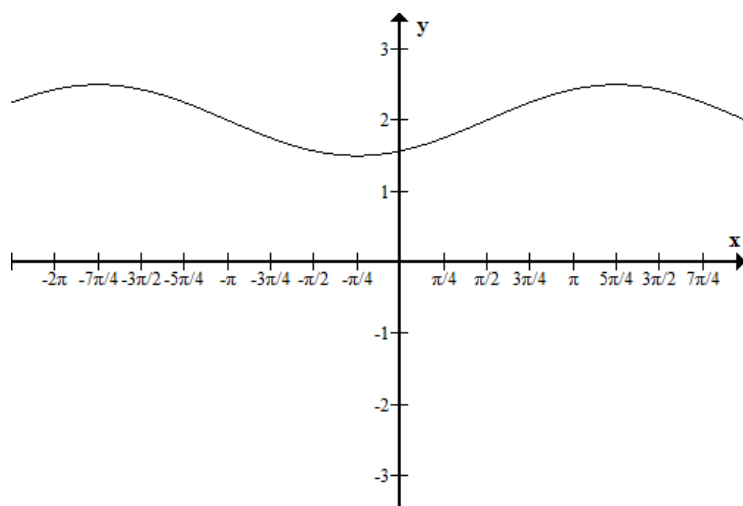


## Handout D- Determining Trig Function Equations

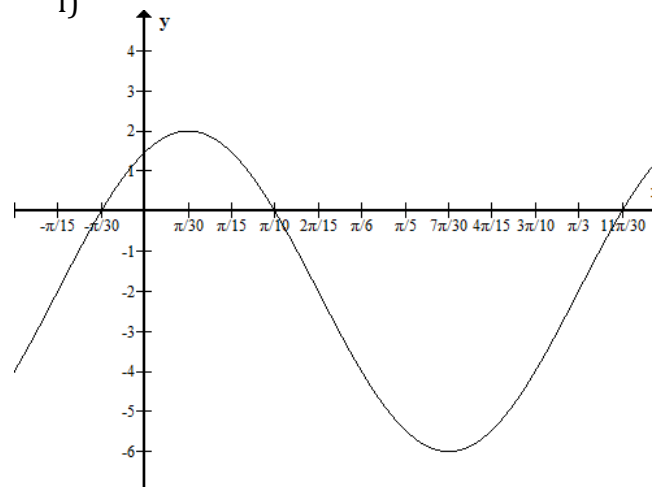
1. State two equations –(one in terms of  $\sin \vartheta$ , and one in terms of  $\cos \vartheta$ ) that could represent the given graph. Use the minimum *positive* phase shift possible.



e)



f)



2. Graph each reciprocal trig functions:

a)  $y = \sec 3\left(\theta - \frac{\pi}{3}\right)$

b)  $y = -\csc\left(\frac{3}{2}x - \pi\right) + 1$

c)  $y = \csc 3\left(x + \frac{1}{2}\pi\right) + 3$

d)  $y = -\sec(4x) + \pi$

**Solutions** 1a)  $y = -2\cos x + 1$ ;  $y = 2\sin\left(x - \frac{\pi}{2}\right) + 1$  1b)  $y = 2\sin 3\left(x - \frac{\pi}{6}\right)$ ;  $y = -2\cos 3x$ 

1c)  $y = 3\sin\left(x - \frac{\pi}{4}\right)$ ;  $y = 3\cos\left(x - \frac{3\pi}{4}\right)$  1d)  $y = 3\sin\frac{1}{2}\left(x - \frac{\pi}{3}\right)$ ;  $y = 3\cos\frac{1}{2}\left(x - \frac{4\pi}{3}\right)$

1e)  $y = \frac{1}{2}\sin\frac{2}{3}\left(x - \frac{\pi}{2}\right) + 2$ ;  $y = \frac{1}{2}\cos\frac{2}{3}\left(x - \frac{5\pi}{4}\right) + 2$  1f)  $y = 4\sin 5\left(x - \frac{\pi}{10}\right) - 2$ ;  $y = 4\cos 5\left(x - \frac{\pi}{30}\right) - 2$

**Solutions #2:**