Homework From Tuesday

Alex De Biasio

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AM 13.6 Sigma Notation

Write the series in expanded form.

Problem 7 $\sum_{t=-2}^{2} 4^t$.

$4^{-2} + 4^{-1} + 4^0 + 4^1 + 4^2$

In Exercises 9-16, express the series using sigma notation.

Problem 9 $4 + 8 + 12 + 16 + 20$

$\sum_{n=1}^{5} 4n$

Problem 11 $5 + 9 + 13 + \cdots + 101$

$\sum_{n=1}^{25} (4n + 1)$

Problem 13 $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$

Problem 15 $\sin x + \sin 2x + \sin 3x + \dots$

$\sum_{n=1}^{\infty} \sin(nx)$

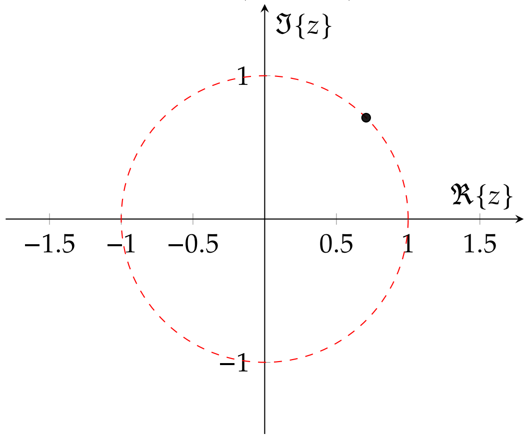
Problem 17 Show that $\sum_{t=1}^{4} \log t = \log 24$.

We know that $\sum_{t=1}^{4} \log t = \log 1 + \log 2 + \log 3 + \log 4$. When you add logarithms that have the same base, you multiply their arguments, which means that this equals $\log(1 \cdot 2 \cdot 3 \cdot 4)$. So, it is equal to $\log 24$.

Problem 19 Evaluate:

(a) $\sum_{n=1}^{8}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^n$

Note that the point $\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$ is on the unit circle in the complex plane:

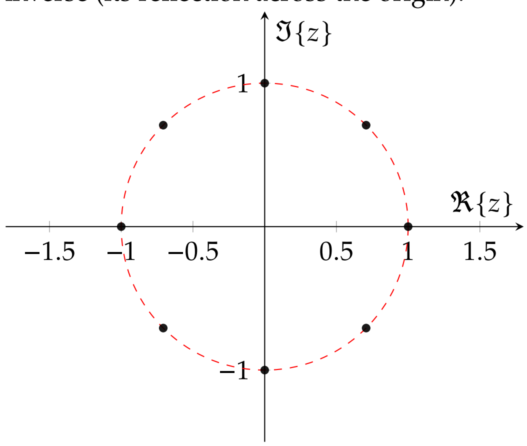


A curious property. As a point on the unit circle in the complex plane, when raised to a power, is just a rotation of itself, I suspect that this can help us with the problem.

We also know that it has an angle of $\pi/4$ from the point $(1,0)$ on the unit circle, as it is the terminal point of $\pi/4$. Thus, we can easily raise it to powers using exponential form:

Power	Exponential form
1	$e^{\pi i/4}$
2	$e^{\pi i/2}$
3	$e^{3\pi i/4}$
...	...
8	1

Graphing these on the complex plane, we can see that each point has an inverse (its reflection across the origin):



Thus, our sum is $\boxed{0}$, as everything cancels out.

(b) $\sum_{n=1}^{8} \left| \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right)^n \right|$

We know that the absolute value of a complex number is just its magnitude. If each point is on the unit circle, each point must have a magnitude of 1, and we know the answer must be $\boxed{8}$ (as there are 8 points).

Express the given series using sigma notation

Problem 25

(a) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32}$

We can see that the common ratio is $-\frac{1}{2}$, so the nth term is equal to $(-1/2)^n$. Our answer is $\boxed{\sum_{n=1}^{6} (-1/2)^n}$.

(b) $-1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}$

This is just a negation of part **(a)**. We have $\boxed{\sum_{n=1}^{6} -(-1/2)^n}$.

Problem 27 $1 - 3 + 5 - 7 + \cdots - 99$

Here, there are two arithmetic series:

$$1 + 5 + 9 + \cdots + 97 \tag{1}$$
$$-(3 + 7 + 11 + \cdots + 99) \tag{2}$$

To find the number of terms in each, we can transform it into the form $1 + 2 + 3 + \cdots + n$, which reveals that there are n terms.

- For *(1)*, we can add 3 to each term to get $4, 8, 12, \ldots, 100$, then divide by 4 to get $1, 2, 3, \ldots, 25$.
- For *(2)*, we can subtract 1 from each term to get $-4, -8, -12, \ldots, -100$, then divide by -4 to get $1, 2, 3, \ldots, 25$.

Thus, both series have 25 terms. We can write it as

$$\sum_{n=1}^{25}(4n-3) - \sum_{m=1}^{25}(4n-1)$$

Using distribution, this is equal to

$$\sum_{n=1}^{25}(4n) - \sum_{n=1}^{25}(3) - \sum_{n=1}^{25}(4n) + \sum_{n=1}^{25}(1),$$

and the $\sum_{n=1}^{25}(4n)$'s cancel to leave us with

$$-\sum_{n=1}^{25}(3) + \sum_{n=1}^{25}(1).$$

Recombine the sums to get

$$\sum_{n=1}^{25}(-2),$$

which is equal to $-2 \cdot 25 = \boxed{\text{-50}}$.