

## Faculty

**L. Capogna**, Professor and Head; Ph.D., Purdue University, 1996. Partial differential equations.

**J. Abraham**, Professor of Practice and Actuarial Mathematics Coordinator; Fellow, Society of Actuaries, 1991; B.S., University of Iowa, 1980.

**A. Arnold**, Assistant Professor; Ph.D., Case Western University, 2014. Mathematical biology, bayesian inference, parameter estimation in biological systems.

**F. Bernardi**, Assistant Professor; Ph.D., University of North Carolina at Chapel Hill, 2018. Small-scale fluid mechanics and microfluidics, in particular modeling particle transport and water filtration systems.

**M. Blais**, Teaching Associate Professor, Coordinator of Professional Science Master's Programs and Associate Department Head; Ph.D., Cornell University, 2005. Mathematical finance.

**J. D. Fehribach**, Professor; Ph.D., Duke University, 1985. Partial differential equations and scientific computing, free and moving boundary problems (crystal growth), nonequilibrium thermodynamics and averaging (molten carbonate fuel cells).

**J. Goulet**, Teaching Professor and Coordinator, Master of Mathematics for Educators Program; Ph.D., Rensselaer Polytechnic Institute, 1976. Applications of linear algebra, cross departmental course development, project development, K-12 relations with colleges, mathematics of digital and analog sound and music.

**A. C. Heinricher**, Professor; Ph.D., Carnegie Mellon University, 1986. Applied probability, stochastic processes and optimal control theory.

**M. Humi**, Professor; Ph.D., Weizmann Institute of Science, 1969. Mathematical physics, applied mathematics and modeling, Lie groups, differential equations, numerical analysis, turbulence and chaos.

**M. Johnson**, Teaching Associate Professor; Ph.D., Clark University 2012. Industrial organization, game theory.

**C. J. Larsen**, Professor; Ph.D., Carnegie Mellon University, 1996. Variational problems from applications such as optimal design, fracture mechanics, and image segmentation, calculus of variations, partial differential equations, geometric measure theory, analysis of free boundaries and free discontinuity sets.

**R. Y. Lui**, Professor; Ph.D., University of Minnesota, 1981. Mathematical biology, partial differential equations.

**K. A. Lurie**, Professor; Ph.D., 1964, D.Sc., 1972, A. F. Ioffe Physical-Technical Institute, Academy of Sciences of the USSR, Russia. Control theory for distributed parameter systems, optimization and nonconvex variational calculus, optimal design.

**W. J. Martin**, Professor; Ph.D., University of Waterloo, 1992. Algebraic combinatorics, applied combinatorics.

**U. Mosco**, H. J. Gay Professor; Libera Docenza, University of Rome, 1967. Partial differential equations, convex analysis, optimal control, variational calculus, fractals.

**B. Nandram**, Professor; Ph.D., University of Iowa, 1989. Survey sampling theory and methods, Bayes and empirical Bayes theory and methods, categorical data analysis.

**P. O'Cathain**, Assistant Professor; Ph.D., National University of Ireland, Galway, 2012. Combinatorics, algebra and compressed sensing.

**S. Olson**, Associate Professor; Ph.D., North Carolina State University 2008. Mathematical biology, computational biofluids, scientific computing.

**R. C. Paffenroth**, Associate Professor; Ph.D., University of Maryland, 1999. Large scale data analytics, statistical machine learning, compressed sensing, network analysis.

**B. Peiris**, Assistant Teaching Professor, Ph.D., Southern Illinois University, Carbondale, 2014. Bayesian Statistics, order restricted inference, meta-analysis.

**G. Peng**, Assistant Professor; Ph.D., Purdue University, 2014. Partial differential equations with a focus on applications to the sciences.

**B. Posterro**, Teaching Associate Professor; M.S., Financial Mathematics, Worcester Polytechnic Institute, 2010, M.S. Applied Mathematics, Worcester Polytechnic Institute, 2000.

**A. Sales**, Assistant Professor; Ph.D., University of Michigan, 2013. Methods for causal inference using administrative or high-dimensional data, especially in education.

**M. Sarkis**, Professor; Ph.D., Courant Institute of Mathematical Sciences, 1994. Domain decomposition methods, numerical analysis, parallel computing, computational fluid dynamics, preconditioned iterative methods for linear and non-linear problems, numerical partial differential equations, mixed and non-conforming finite methods, overlapping non-matching grids, mortar finite elements, eigenvalue solvers, aeroelasticity, porous media reservoir modeling.

**B. Servatius**, Professor; Ph.D., Syracuse University, 1987. Combinatorics, matroid and graph theory, structural topology, geometry, history and philosophy of mathematics.

**S. Sturm**, Associate Professor; Ph.D., TU Berlin 2010. Mathematical finance: stochastic volatility, optimal portfolio problems, systemic risk; stochastic analysis: backward stochastic differential equations, large deviations, Malliavin calculus.

**D. Tang**, Professor; Ph.D., University of Wisconsin, 1988. Biofluids, biosolids, blood flow, mathematical modeling, numerical methods, scientific computing, nonlinear analysis, computational fluid dynamics.

**B. S. Tilley**, Associate Professor; Ph.D., Northwestern University, 1994. Free-boundary problems in continuum mechanics, interfacial fluid dynamics, viscous flows, partial differential equations, mathematical modeling, asymptotic methods.

**B. Vernescu**, Professor; Ph.D., Institute of Mathematics, Bucharest, Romania, 1989. Partial differential equations, phase transitions and free-boundaries, viscous flow in porous media, asymptotic methods and homogenization.

**D. Volkov**, Associate Professor; Ph.D., Rutgers University, 2001. Electromagnetic waves, inverse problems, wave propagation in waveguides and in periodic structures, electrified fluid jets.

**G. Wang**, Assistant Professor; Ph.D., Boston University, 2013. Stochastic control, mathematical finance, stochastic analysis, applied probability.

**S. Weekes**, Professor; Ph.D., University of Michigan, 1995. Numerical analysis, computational fluid dynamics, porous media flow, hyperbolic conservation laws, shock capturing schemes.

**M. Wu**, Visiting Assistant Professor; Ph.D., University of California, Irvine, 2012. Mathematical biology, modeling of living systems.

**Z. Wu**, Associate Professor; Ph.D., Yale University, 2009. Biostatistics, high-dimensional model selection, linear and generalized linear modeling, statistical genetics, bioinformatics.

**V. Yakovlev**, Research Associate; Ph.D., Institute of Radio Engineering and Electronics, Russian Academy of Sciences, 1991. Antennas for MW and MMW communications, electromagnetic fields in transmission lines and along media interfaces, control and optimization of electromagnetic and temperature fields in microwave thermal processing, issues in modeling of microwave heating, computational electromagnetics with neural networks, numerical methods, algorithms and CAD tools for RF, MW and MMW components and subsystems.

**Z. Zhang**, Associate Professor; Ph.D., Brown University, 2014, Shanghai University, 2011. Numerical analysis, scientific computing, computational and applied mathematics, uncertainty qualification.

**J. Zou**, Associate Professor; Ph.D., University of Connecticut, 2009. Financial time series (especially high frequency financial data), spatial statistics, biosurveillance, high dimensional statistical inference, Bayesian statistics.

### **Emeritus**

**P. W. Davis**, Professor

**W. J. Hardell**, Professor

**J. J. Malone**, Professor

**B. C. McQuarrie**, Professor

**W. B. Miller**, Professor

**D. Vermes**, Professor

## **Research Interests**

Active areas of research in the Mathematical Sciences Department include applied and computational mathematics, industrial mathematics, applied statistics, scientific computing, numerical analysis, ordinary and partial differential equations, non-linear analysis, electric power systems, control theory, optimal design, composite materials, homogenization, computational fluid dynamics, biofluids, dynamical systems, free and moving boundary problems, porous media modeling, turbulence and chaos, mathematical physics, mathematical biology, operations research, linear and nonlinear programming, discrete mathematics, graph theory, group theory, linear algebra, combinatorics, applied probability, stochastic processes, time series analysis, Bayesian statistics, Bayesian computation, survey research methodology, categorical data analysis, Monte Carlo methodology, statistical computing, survival analysis and model selection.

## **Programs of Study**

The Mathematical Sciences Department offers four programs leading to the degree of master of science, a combined B.S./Master's program, a program leading to the degree of master of mathematics for educators, and a program leading to the degree of doctor of philosophy.

### **Master of Science in Applied Mathematics Program**

This program gives students a broad background in mathematics, placing an emphasis on areas with the highest demand in applications: numerical methods and scientific computation, mathematical modeling, discrete mathematics, mathematical materials science, optimization and operations research. In addition to these advanced areas of specialization, students are encouraged to acquire breadth by choosing elective courses in other fields that complement their studies in applied mathematics. Students have a choice of completing their master's thesis or project in cooperation with one of the department's established industrial partners. The program provides a suitable foundation for the pursuit of a Ph.D.

degree in applied mathematics or a related field, or for a career in industry immediately after graduation.

### **Master of Science in Applied Statistics Program**

This program gives graduates the knowledge and experience to tackle problems of statistical design, analysis and control likely to be encountered in business, industry or academia. The program is designed to acquaint students with the theory underlying modern statistical methods, to provide breadth in diverse areas of statistics and to give students practical experience through extensive application of statistical theory to real problems.

Through the selection of elective courses, the student may choose a program with an industrial emphasis or one with a more theoretical emphasis.

### **Professional Master of Science in Financial Mathematics Program**

This program offers an efficient, practice-oriented track to prepare students for quantitative careers in the financial industry, including banks, insurance companies, and investment and securities firms. The program gives students a solid background and sufficient breadth in the mathematical and statistical foundations needed to understand the cutting edge techniques of today and to keep up with future developments in this rapidly evolving area over the span of their careers. It also equips students with expertise in quantitative financial modeling and the computational methods and skills that are used to implement the models. The mathematical knowledge is complemented by studies in financial management, information technology and/or computer science.

The bridge from the academic environment to the professional workplace is provided by a professional master's project that involves the solution of a concrete, real-world problem directly originating in the financial industry. Students are encouraged to complete summer internships at financial firms. The department may help students to find suitable financial internships through the industrial connections of faculty

affiliated with the Center for Industrial Mathematics and Statistics. Graduates of the program are expected to start or advance their professional careers in such areas as financial product development and pricing, risk management, investment decision support and portfolio management.

### **Professional Master of Science in Industrial Mathematics Program**

This is a practice-oriented program that prepares students for successful careers in industry. The graduates are expected to be generalized problem-solvers, capable of moving from task to task within an organization. In industry, mathematicians need not only the standard mathematical and statistical modeling and computational tools, but also knowledge within other areas of science or engineering. This program aims at developing the analytical, modeling and computational skills needed by mathematicians who work in industrial environments. It also provides the breadth required by industrial multidisciplinary team environments through courses in one area of science or engineering, e.g., physics, computer science, mechanical engineering, and electrical and computer engineering.

The connection between academic training and industrial experience is provided by an industrial professional master's project that involves the solution of a concrete, real-world problem originating in industry. The department, through the industrial connections of the faculty affiliated with the Center for Industrial Mathematics and Statistics, may help students identify and select suitable industrial internships. Graduates of the program are expected to start or advance their professional careers in industry.

### **Master of Mathematics for Educators (MME)**

This is an evening and/or online program designed primarily for secondary school mathematics teachers. Courses offer a solid foundation in areas such as geometry, algebra, modeling, discrete math and statistics, while also including the study of modern applications. Additionally, students develop materials, based on coursework, which may be used in their classes. Technology is introduced when

possible to give students exposure for future consideration. Examples include Geometer's Sketchpad; Maple for algebra, calculus and graphics; Matlab for analysis of sound and music; and the TI CBL for motion and heat.

### **Master of Science in Mathematics for Educators (MMED)**

The Master of Science in Mathematics for Educators is designed specifically for middle school, high school and junior college in-service educators. The emphasis of the program is put on mathematics content coursework combined with courses in assessment and evaluation theory and a culminating project designed by the participant. The mathematics content courses, designed for educators, offer teachers a solid foundation in areas such as geometry, algebra, modeling, discrete math and statistics, while also including the study of modern applications. In these courses, participants have the opportunity to develop materials, based on coursework, which may be used in their classes. Throughout the courses, technology is introduced whenever possible to help educators become familiar with the options available for use in the classroom. Examples of this include Geometer's Sketchpad and the TI CBL for motion and heat. This combination of content courses, assessment and evaluation theory courses, and a final project are perfect for educators looking for a program that emphasizes mathematics and supports educators in learning how to better evaluate their effectiveness in the classroom. For information about admissions and requirements, see the listing under STEM for Educators.

### **Doctor of Philosophy in Mathematical Sciences Program**

The goal of this program is to produce active and creative problem solvers, capable of contributing in academic and industrial environments. One distinguishing feature of this program is an optional Ph.D. project to be completed under the guidance of an external sponsor, e.g., from industry or a national research center. The intention of this project is to connect theoretical knowledge with relevant applications and to improve skills in applying and communicating mathematics.

### **Doctor of Philosophy in Statistics Program**

The overall objective is to create a highly competitive program that produces future scholars and leaders in Statistics. The program will provide rigorous and comprehensive training in mathematics, statistics and related areas, as well as in critical thinking and problem solving for statistical challenges in data-related researches and applications. The goal is to prepare future leading statisticians in academia, industry, and government.

### **Combined B.S./Master's Program**

This program allows a student to work concurrently toward bachelor and master of science degrees in applied mathematics, applied statistics, financial mathematics and industrial mathematics.

### **Admission Requirements**

A basic knowledge of undergraduate analysis, linear algebra and differential equations is assumed for applicants to the master's programs in applied mathematics and industrial mathematics. A strong background in mathematics, which should include courses in undergraduate analysis and linear algebra, is assumed for applicants to the master's program in financial mathematics. Typically, an entering student in the master of science in applied statistics program will have an undergraduate major in the mathematical sciences, engineering or a physical science; however, individuals with other backgrounds will be considered. In any case, an applicant will need a strong background in mathematics, which should include courses in undergraduate analysis and probability. Students with serious deficiencies may be required to correct them on a noncredit basis. Applicants to the Mathematical Sciences Ph.D. Program should submit GRE Mathematics Subject Test scores if possible; an applicant who finds it difficult to submit a score is welcome to contact the Mathematical Sciences Department Graduate Admissions Committee (ma-questions@wpi.edu) to discuss the applicant's situation.

For the applicants to the Ph.D. Program in Statistics, strong background of undergraduate analysis, linear algebra



and probability is assumed; the GRE Mathematics Subject Test is recommended but not required.

Candidates for the master of mathematics for educators degree must have a bachelor's degree and must possess a background equivalent to at least a minor in mathematics, including calculus, linear algebra, and statistics. Students are encouraged to enroll in courses on an ad hoc basis without official program admission. However, (at most) four such courses may be taken prior to admission.

## Degree Requirements

### For the M.S. in Applied Mathematics

The master's program in Applied Mathematics requires a minimum of 30 credit-hours of coursework. Additional credit from coursework may be required by the department depending on the student's background. The student's program must include at least seven MA numbered courses other than 501 or 511. Among these must be MA 503, MA 510, and either MA 535 or MA 530. In addition, students are required to complete a Capstone Experience, which can be satisfied by one of the following options:

- A six credit master's thesis.
- A three to six credit master's project.
- A three credit master's practicum.
- A three credit research review report or research proposal.
- A master's exam.

The master's thesis is an original piece of mathematical research work which focuses on advancing the state of the mathematical art. The master's project consists of a creative application of mathematics to a real-world problem. It focuses on problem definition and solution using mathematical tools. The master's practicum requires a student to demonstrate the integration of advanced mathematical concepts and methods into professional practice. This could be done through a summer internship in industry or an applied research laboratory.

The remaining courses may be chosen from the graduate offerings of the Mathematical Sciences Department.

Upper-level undergraduate mathematics courses or a two-course graduate sequence in another department may be taken for graduate credit, subject to the approval of the departmental Graduate Committee. Candidates are required to successfully complete the graduate seminar MA 557.

### For the M.S. in Applied Statistics

The master's program in Applied Statistics requires a minimum of 30 credit-hours of coursework. Additional credit from coursework may be required by the department depending on the student's background. Courses taken must include MA 540, MA 541, MA 546, MA 547, 3 credits of MA 559 and at least three additional departmental statistics offerings: MA 509 and courses numbered 542 through 556. Students who can demonstrate a legitimate conflict in scheduling MA 559 will be assigned an alternative activity by the Mathematical Science Department Graduate Committee. In addition the student must complete a Capstone Experience, which can be satisfied by one of the following options:

- A six credit master's thesis.
- A three to six credit master's project.
- A three credit master's practicum.
- A three credit research review report or research proposal.
- A master's exam.

Upper-level undergraduate courses may be taken for graduate credit subject to the approval of the departmental Graduate Committee.

### For the M.S. in Financial Mathematics

The master's program in Financial Mathematics requires a minimum of 30 credit-hours of coursework. Additional credit from coursework may be required by the department depending on the student's background. The curriculum consists of the following components:

#### 1. 6 credits from required foundation courses:

MA 529 Stochastic Processes **or**  
MA 503 Lebesgue Measure and Integration  
MA 528 Measure Theoretic Probability Theory **or**  
MA 540 Probability and Mathematical Statistics I

#### 2. 12 credits from core financial mathematics courses:

MA 571 Financial Mathematics I  
MA 572 Financial Mathematics II  
MA 573 Computational Methods of Financial Mathematics  
MA 574 Portfolio Valuation and Risk Management  
MA 575 Market and Credit Risk Models and Management

#### 3. 3 credits chosen from Mathematical Sciences graduate courses MA 502-590.

B.S./M.S. students can count undergraduate courses MA 4213 Risk Theory, MA 4214 Survival Models, MA 4235 Mathematical Optimization, MA 4237 Probabilistic Methods in Operations Research, MA 4451 Boundary Value Problems, MA 4473 Partial Differential Equations, MA 4632 Probability and Mathematical Statistics II towards electives

#### 4. 6 credit block in one of the following complementary areas outside of the Mathematical Sciences Department: Financial Management, Information Technology, or Computer Science.

Students with a degree or substantial work experience in one of the above complementary areas can substitute them with other courses subject to prior approval by the graduate committee

B.S./M.S. students can count suitable undergraduate courses towards the complementary area requirement according the number of credits of the corresponding graduate courses

2 of the complementary area credits can be earned by taking MA 579 Financial Programming Workshop

#### 5. Capstone Project, which may be satisfied by one of the following options:

- A three to six credit master's project.
- A three credit master's practicum.
- A three credit capstone course in financial mathematics.

The master's project consists of a creative application of mathematics to a real-world problem originating in the financial industry. It focuses on problem definition and solution using mathematical tools. The master's practicum requires a student to demonstrate the integration of advanced mathematical concepts and methods into professional practice. This could be done through an approved summer internship in industry or an applied research laboratory. The capstone course in financial mathematics can be chosen from MA 572, MA 573, MA 574, or MA 575 and will be an enhanced version of the course with extra work assigned. Prior to the start of the capstone course, a student seeking to use the course to satisfy the requirement must declare this intention to the professor of the course.

#### 6. MA 562A and MA 562B Professional Master's Seminar (for no credit)

### For the M.S. in Industrial Mathematics

The master's program in Industrial Mathematics requires a minimum of 30 credit-hours of coursework. Additional credit from coursework may be required by the department depending on the student's background. Students must complete four foundation courses: MA 503, MA 510 and two courses out of MA 508, MA 509, MA 529 and MA 530. Students must also complete a 12-credit-hour module composed of two courses within the department and a sequence of two courses from one graduate program outside the Mathematical Sciences Department. The department offers a wide selection of modules to suit students' interest and expertise.

In addition, students are required to complete a 3-credit-hour elective from the Mathematical Sciences Department and a 3-credit-hour master's project on a problem originating from industry. Candidates are required to successfully complete the Professional Master's Seminars MA 562A and MA 562B. The Plan of Study and the project topic require prior approval by the departmental Graduate Committee.

### Examples of Modules for the M.S. Degree in Industrial Mathematics

The courses comprising the 12-credit module should form a coherent sequence that provides exposure to an area outside of mathematics and statistics, providing at the same time the mathematical tools required by that particular area. Examples of typical modules are:

- Dynamics and control module—MA 512, MA 540, ME 5220, 5221, 5222, 5223;
- Materials module—MA 512, MA 526, and ME 5311;
- Fluid dynamics module—MA 512, MA 526, ME 511 and ME 5101, 5102, 5103;
- Biomedical engineering module—MA 512, MA 526, BME/ME 550 and BME/ME 552;
- Machine learning module—MA 540, MA 541, CS 509 and CS 539;
- Cryptography module—MA 533, MA 514, CS 503 and ECE 578.

### For the Combined B.S./Master's Programs in Applied Mathematics and Applied Statistics

Credits from no more than four courses may be counted toward both the undergraduate and graduate degrees. All of these courses must be 4000-level or above, and at least one must be a graduate course. Three of them must be beyond the 7 units of mathematics required for the B.S. degree. Additionally, students are advised that all requirements of a particular master's program must be satisfied in order to receive the degree, and these courses should be selected accordingly.

Acceptance into the program means that the candidate is qualified for graduate school and signifies approval of the four courses to be counted for credit toward both degrees. However, in order to obtain both undergraduate and graduate credit for these courses, grades of B or better have to be obtained.

### For the Master of Mathematics for Educators (M.M.E.)

Candidates for the Master of Mathematics for educators must successfully complete 30 credit hours of graduate study, including a 6-credit-hour project (see MME 592, MME 594, MME 596). This

project will typically consist of a classroom study within the context of a secondary mathematics course and will be advised by faculty in the Mathematical Sciences Department. Typically, a student will enroll in 4 credit hours per semester during the fall and spring, with the remaining credit hours taken in the summer.

Students may complete the degree in as little as slightly over two years by taking two courses per semester, 3 semesters per year, and doing a project. However, the program can accommodate other completion schedules as well. The MME degree may be used to satisfy the Massachusetts Professional License requirement, provided the person holds an Initial License.

### For the Master of Science in Mathematics for Educators (MMED)

For a complete overview of degree requirements, please see STEM for Educators.

### For the Ph.D.

The course of study leading to the doctor of philosophy in mathematical science and the doctor of philosophy in statistics requires the completion of at least 90 credit hours beyond the bachelor's degree or at least 60 credit hours beyond the master's degree, as follows:

General Courses (credited for students with master's degrees)	30 credits
Research Preparation Phase	24-30 credits
Research-Related Courses or Independent Studies	9-18 credits
Ph.D. Project	1-9 credits
Extra-Departmental Studies	6 credits
Dissertation Research	at least 30 credits

A brief description of other Ph.D. program requirements follows below. For further details, students are advised to consult the document *Ph.D. Program Requirements and Administrative Rules for the Department of Mathematical Sciences*, available from the departmental graduate secretary.

Within a full-time student's first semester of study (second semester for part-time students), a Plan of Study leading to the Ph.D. degree must be submitted to the departmental Graduate Committee for review and approval. The Plan of Study may subsequently be modified with review by the departmental Graduate Committee.

### **Extra-Departmental Studies Requirement**

A student must complete at least six semester hours of courses, 500 level or higher, in WPI departments other than the Mathematical Sciences Department.

### **General Comprehensive Examination**

A student must pass the general comprehensive examination (GCE) in order to become a Ph.D. candidate. The purpose of the GCE is to determine whether a student possesses the fundamental knowledge and skills necessary for study and research at the Ph.D. level. It is a written examination offered three times a year, once each in January, May, and August. A student must pass by January of their second year if they enter in the fall, and May of their second year if they enter in the spring.

### **Mathematical Sciences Ph.D. Project**

A student may complete a Ph.D. project involving a problem originating with a sponsor external to the department. The purposes of the project are to broaden perspectives on the relevance and applications of mathematics and to improve skills in communicating mathematics and formulating and solving mathematical problems. Students are encouraged to work with industrial sponsors on problems involving applications of the mathematical sciences. Each Ph.D. project requires prior approval by the project advisor, the external sponsor, and the departmental Graduate Committee.

### **Ph.D. Preliminary Examination**

Successful completion of the preliminary examination is required before a student can register for dissertation research credits. The purpose of the preliminary examination is to determine whether a student's understanding of advanced areas of mathematics is adequate to conduct independent research and successfully complete a dissertation. The preliminary examination consists of both written and oral parts. A full-time student must make the first attempt by the end of his or her third year (sixth year for part-time students) in the Ph.D. program.

### **Ph.D. Dissertation**

The Ph.D. dissertation is a significant work of original research conducted under the supervision of a dissertation advisor, who is normally a member of the departmental faculty. The dissertation advisor chairs the student's dissertation committee, which consists of at least five members, including one recognized expert external to the department, and which must be approved by the departmental Graduate Committee. At least six months prior to completion of the dissertation, a student must submit a written dissertation proposal and present a public seminar on the research plan described in the proposal. The proposal must be approved by the dissertation committee. Upon completion of the dissertation and other program requirements, the student presents the dissertation to the dissertation committee and to the general community in a public oral defense. The dissertation committee determines whether the dissertation is acceptable.

### **Unsatisfactory Progress**

If the aforementioned milestones are not met, then the student must petition the graduate program committee to request extra time to meet the requirements or the student will no longer be part of the Ph.D. program as of the following semester.

## **Mathematical Sciences Computer Facilities**

The Mathematical Sciences Department makes up-to-date computing equipment available for use by students in its programs.

Current facilities include a mixed environment of approximately 85 Windows, Linux/Unix and Macintosh workstations utilizing the latest in single- and dual-processor 32 and 64 bit technology as well as 4 Bloomberg terminals. Access is available to our supercomputer, a 16 CPU SGI Altix 350. The Mathematical Sciences Department also has 3 state-of-the-art computer labs, one each dedicated to the Calculus, Statistics, and Financial Mathematics programs.

The department is continually adding new resources to give our faculty and students the tools they need as they advance in their research and studies.

## **Center for Industrial Mathematics and Statistics (CIMS)**

[www.wpi.edu/+CIMS](http://www.wpi.edu/+CIMS)

The Center for Industrial Mathematics and Statistics was established in 1997 to foster partnerships between the university and industry, business and government in mathematics and statistics research.

The problems facing business and industry are growing ever more complex, and their solutions often involve sophisticated mathematics. The faculty members and students associated with CIMS have the expertise to address today's complex problems and provide solutions that use relevant mathematics and statistics.

The Center offers undergraduates and graduate students the opportunity to gain real-world experience in the corporate world through projects and internships that make them more competitive in today's job market. In addition, it helps companies address their needs for mathematical solutions and enhances their technological competitiveness.

The industrial projects in mathematics and statistics offered by CIMS provide a unique education for successful careers in industry, business and higher education.

## **Course Descriptions**

All courses are 3 credits unless otherwise noted.

### **Mathematical Sciences**

#### **MA 500. Basic Real Analysis**

This course covers basic set theory, topology of  $\mathbb{R}^n$ , continuous functions, uniform convergence, compactness, infinite series, theory of differentiation and integration. Other topics covered may include the inverse and implicit function theorems and Riemann-Stieltjes integration. Students may not count both MA 3831 and MA 500 toward their undergraduate degree requirements.

#### **MA 501. Engineering Mathematics**

This course develops mathematical techniques used in the engineering disciplines. Preliminary concepts will be reviewed as necessary, including vector spaces, matrices and eigenvalues. The principal topics covered will include vector calculus, Fourier transforms, fast Fourier transforms and Laplace transformations. Applications of these



techniques for the solution of boundary value and initial value problems will be given. The problems treated and solved in this course are typical of those seen in applications and include problems of heat conduction, mechanical vibrations and wave propagation. (Prerequisite: A knowledge of ordinary differential equations, linear algebra and multivariable calculus is assumed.)

### **MA 502. Linear Algebra**

This course provides an introduction to the theory and methods of applicable linear algebra. The goal is to bring out the fundamental concepts and techniques that underlie and unify the many ways in which linear algebra is used in applications. The course is suitable for students in mathematics and other disciplines who wish to obtain deeper insights into this very important subject than are normally offered in undergraduate courses. It is also intended to provide a foundation for further study in subjects such as numerical linear algebra and functional analysis.

### **MA 503. Lebesgue Measure and Integration**

This course begins with a review of topics normally covered in undergraduate analysis courses: open, closed and compact sets;  $\liminf$  and  $\limsup$ ; continuity and uniform convergence. Next the course covers Lebesgue measure in  $\mathbb{R}^n$  including the Cantor set, the concept of a  $\sigma$ -algebra, the construction of a nonmeasurable set, measurable functions, semicontinuity, Egorov's and Lusin's theorems, and convergence in measure. Next we cover Lebesgue integration, integral convergence theorems (monotone and dominated), Tchebyshev's inequality and Tonelli's and Fubini's theorems. Finally  $L^p$  spaces are introduced with emphasis on  $L^2$  as a Hilbert space. Other related topics will be covered at the instructor's discretion. (Prerequisite: Basic knowledge of undergraduate analysis is assumed.)

### **MA 504. Functional Analysis**

This course will give a comprehensive presentation of fundamental concepts and theorems in Banach and Hilbert spaces. Whenever possible, the theory will be illustrated by examples in Lebesgue spaces. Topics include: The Hahn-Banach theorems, the Uniform Boundedness principle (Banach-Steinhaus Theorem), the Open Mapping and Closed Graph theorems, and weak topologies and convergence. Additional topics will be covered at the instructor's discretion. (Prerequisite: MA 503 or equivalent.)

### **MA 505. Complex Analysis**

This course will provide a rigorous and thorough treatment of the theory of functions of one complex variable. The topics to be covered include complex numbers, complex differentiation, the Cauchy-Riemann equations, analytic functions, Cauchy's theorem, complex integration, the Cauchy integral formula, Liouville's theorem, the Gauss mean value theorem, the maximum modulus theorem, Rouché's theorem, the Poisson integral formula, Taylor-Laurent expansions, singularity theory, conformal mapping with

applications, analytic continuation, Schwarz's reflection principle and elliptic functions. (Prerequisite: knowledge of undergraduate analysis.)

### **MA 508. Mathematical Modeling**

This course introduces mathematical model-building using dimensional analysis, perturbation theory and variational principles. Models are selected from the natural and social sciences according to the interests of the instructor and students. Examples are: planetary orbits, spring-mass systems, fluid flow, isomers in organic chemistry, biological competition, biochemical kinetics and physiological flow. Computer simulation of these models will also be considered. (Prerequisite: knowledge of ordinary differential equations and of analysis at the level of MA 501 is assumed.)

### **MA 509. Stochastic Modeling**

This course gives students a background in the theory and methods of probability, stochastic processes and statistics for applications. The course begins with a brief review of basic probability, discrete and continuous random variables, expectations, conditional probability and basic statistical inference. Topics covered in greater depth include generating functions, limit theorems, basic stochastic processes, discrete and continuous time Markov chains, and basic queueing theory including M/M/1 and M/G/1 queues. (Prerequisite: knowledge of basic probability at the level of MA 2631 and statistics at the level of MA 2612 is assumed.) This course is offered by special arrangement only, based on expressed student interest.

### **MA 510/CS 522. Numerical Methods**

This course provides an introduction to a broad range of modern numerical techniques that are widely used in computational mathematics, science, and engineering. It is suitable for both mathematics majors and students from other departments. It covers introductory-level material for subjects treated in greater depth in MA 512 and MA 514 and also topics not addressed in either of those courses.

Subject areas include numerical methods for systems of linear and nonlinear equations, interpolation and approximation, differentiation and integration, and differential equations. Specific topics include basic direct and iterative methods for linear systems; classical rootfinding methods; Newton's method and related methods for nonlinear systems; fixed-point iteration; polynomial, piecewise polynomial, and spline interpolation methods; least-squares approximation; orthogonal functions and approximation; basic techniques for numerical differentiation; numerical integration, including adaptive quadrature; and methods for initial-value problems for ordinary differential equations. Additional topics may be included at the instructor's discretion as time permits.

Both theory and practice are examined. Error estimates, rates of convergence, and the consequences of finite precision arithmetic are

also discussed. Topics from linear algebra and elementary functional analysis will be introduced as needed. These may include norms and inner products, orthogonality and orthogonalization, operators and projections, and the concept of a function space. (Prerequisite: knowledge of undergraduate linear algebra and differential equations is assumed, as is familiarity with MATLAB or a higher-level programming language.)

### **MA 511. Applied Statistics for Engineers and Scientists**

This course is an introduction to statistics for graduate students in engineering and the sciences. Topics covered include basic data analysis, issues in the design of studies, an introduction to probability, point and interval estimation and hypothesis testing for means and proportions from one and two samples, simple and multiple regression, analysis of one and two-way tables, one-way analysis of variance. As time permits, additional topics, such as distribution-free methods and the design and analysis of factorial studies will be considered. (Prerequisites: Integral and differential calculus.)

### **MA 512. Numerical Differential Equations**

This course begins where MA 510 ends in the study of the theory and practice of the numerical solution of differential equations. Central topics include a review of initial value problems, including Euler's method, Runge-Kutta methods, multi-step methods, implicit methods and predictor-corrector methods; the solution of two-point boundary value problems by shooting methods and by the discretization of the original problem to form systems of nonlinear equations; numerical stability; existence and uniqueness of solutions; and an introduction to the solution of partial differential equations by finite differences. Other topics might include finite element or boundary element methods, Galerkin methods, collocation, or variational methods. (Prerequisites: graduate or undergraduate numerical analysis. Knowledge of a higher-level programming language is assumed.)

### **MA 514. Numerical Linear Algebra**

This course provides students with the skills necessary to develop, analyze and implement computational methods in linear algebra. The central topics include vector and matrix algebra, vector and matrix norms, the singular value decomposition, the LU and QR decompositions, Householder transformations and Givens rotations, and iterative methods for solving linear systems including Jacobi, Gauss-Seidel, SOR and conjugate gradient methods; and eigenvalue problems. Applications to such problem areas as least squares and optimization will be discussed. Other topics might include: special linear systems, such as symmetric, positive definite, banded or sparse systems; preconditioning; the Cholesky decomposition; sparse tableau and other least-square methods; or algorithms for parallel architectures. (Prerequisite: basic knowledge of linear algebra or equivalent background. Knowledge of a higher-level programming language is assumed.)

### **MA/DS 517. Mathematical Foundations for Data Science**

The foci of this class are the essential statistics and linear algebra skills required for Data Science students. The class builds the foundation for theoretical and computational abilities of the students to analyze high dimensional data sets. Topics covered include Bayes' theorem, the central limit theorem, hypothesis testing, linear equations, linear transformations, matrix algebra, eigenvalues and eigenvectors, and sampling techniques, including Bootstrap and Markov chain Monte Carlo. Students will use these techniques while engaging in hands-on projects with real data. Prerequisites: Some knowledge of integral and differential calculus is recommended.

### **MA 520. Fourier Transforms and Distributions**

The course will cover  $L^1$ ,  $L^2$ ,  $L^\infty$  and basic facts from Hilbert space theory (Hilbert basis, projection theorems, Riesz theory). The first part of the course will introduce Fourier series: the  $L^2$  theory, the  $C^\infty$  theory: rate of convergence, Fourier series of real analytic functions, application to the trapezoidal rule, Fourier transforms in  $L^1$ , Fourier integrals of Gaussians, the Schwartz class  $S$ , Fourier transforms and derivatives, translations, convolution, Fourier transforms in  $L^2$ , and characteristic functions of probability distribution functions. The second part of the course will cover tempered distributions and applications to partial differential equations. Other related topics will be covered at the instructor's discretion. (Prerequisite: MA 503.)

### **MA 521. Partial Differential Equations**

This course considers a variety of material in partial differential equations (PDE). Topics covered will be chosen from the following: classical linear elliptic, parabolic and hyperbolic equations and systems, characteristics, fundamental/Green's solutions, potential theory, the Fredholm alternative, maximum principles, Cauchy problems, Dirichlet/Neumann/Robin problems, weak solutions and variational methods, viscosity solutions, nonlinear equations and systems, wave propagation, free and moving boundary problems, homogenization. Other topics may also be covered. (Prerequisites: MA 503 or equivalent.)

### **MA 522. Hilbert Spaces and Applications to PDE**

The course covers Hilbert space theory with special emphasis on applications to linear ODEs and PDEs. Topics include spectral theory for linear operators in  $n$ -dimensional and infinite dimensional Hilbert spaces, spectral theory for symmetric compact operators, linear and bilinear forms, Riesz and Lax-Milgram theorems, weak derivatives, Sobolev spaces  $H^1$ ,  $H^2$ , Rellich compactness theorem, weak and classical solutions for Dirichlet and Neumann problems in one variable and in  $\mathbb{R}^n$ , Dirichlet variational principle, eigenvalues and eigenvectors. Other related topics will be covered at the instructor's discretion. (Prerequisite: MA 503.)

### **MA 524. Convex Analysis and Optimization**

This course covers topics in functional analysis that are critical to the study of convex optimization problems. The first part of the course will include the minimization theory for quadratic and convex functionals on convex sets and cones, the Legendre-Fenchel duality, variational inequalities and complementarity systems. The second part will include optimal stopping time problems in deterministic control, value functions and Hamilton-Jacobi inequalities and linear and quadratic programming, duality and Kuhn-Tucker multipliers. Other related topics will be covered at the instructor's discretion. (Prerequisite: MA 503.)

### **MA 525. Optimal Control and Design with Composite Materials I**

Modern technology involves a wide application of materials with internal structure adapted to environmental demands. This, the first course in a two-semester sequence, will establish a theoretical basis for identifying structures that provide optimal response to prescribed external factors. Material covered will include basics of the calculus of variations: Euler equations; transversality conditions; Weierstrass-Erdmann conditions for corner points; Legendre, Jacobi and Weierstrass conditions; Hamiltonian form of the necessary conditions; and Noether's theorem. Pontryagin's maximum principle in its original lumped parameter form will be put forth as well as its distributed parameter extension. Chattering regimes of control and relaxation through composites will be introduced at this point. May be offered by special arrangement.

### **MA 526. Optimal Control and Design with Composite Materials II**

Topics presented will include basics of homogenization theory. Bounds on the effective properties of composites will be established using the translation method and Hashin-Shtrikman variational principles. The course concludes with a number of examples demonstrating the use of the theory in producing optimal structural designs. The methodology given in this course turns the problem of optimal design into a problem of rigorous mathematics. This course can be taken independently or as the sequel to MA 525.

### **MA 528. Measure Theoretic Probability Theory**

This course is designed to give graduate students interested in financial mathematics and stochastic analysis the necessary background in measure-theoretic probability and provide a theoretical foundation for Ph.D. students with research interests in analysis and mathematical statistics. Besides classical topics such as the axiomatic foundations of probability, conditional probabilities and independence, random variables and their distributions, and limit theorems, this course focuses on concepts crucial for the understanding of stochastic processes and quantitative finance: conditional expectations, filtrations and martingales as well as change of measure techniques and the Radon-Nikodym theorem. A

wide range of illustrative examples from a topic chosen by the instructor's discretion (e.g. financial mathematics, signal processing, actuarial mathematics) will be presented. (Prerequisite: MA 500 Basic Real Analysis or equivalent.)

### **MA 529. Stochastic Processes**

This course is designed to introduce students to continuous-time stochastic processes. Stochastic processes play a central role in a wide range of applications from signal processing to finance and also offer an alternative novel viewpoint to several areas of mathematical analysis, such as partial differential equations and potential theory. The main topics for this course are martingales, maximal inequalities and applications, optimal stopping and martingale convergence theorems, the strong Markov property, stochastic integration, Ito's formula and applications, martingale representation theorems, Girsanov's theorem and applications, and an introduction to stochastic differential equations, the Feynman-Kac formula, and connections to partial differential equations. Optional topics (at the instructor's discretion) include Markov processes and Poisson and jump-processes. (Prerequisite: MA 528. Measure-Theoretic Probability Theory, which can be taken concurrently (or, with special permission by the instructor, MA 540)).

### **MA 530. Discrete Mathematics**

This course provides the student of mathematics or computer science with an overview of discrete structures and their applications, as well as the basic methods and proof techniques in combinatorics. Topics covered include sets, relations, posets, enumeration, graphs, digraphs, monoids, groups, discrete probability theory and propositional calculus. (Prerequisites: college math at least through calculus. Experience with recursive programming is helpful, but not required.)

### **MA 533. Discrete Mathematics II**

This course is designed to provide an in-depth study of some topics in combinatorial mathematics and discrete optimization. Topics may vary from year to year. Topics covered include, as time permits, partially ordered sets, lattices, matroids, matching theory, Ramsey theory, discrete programming problems, computational complexity of algorithms, branch and bound methods.

### **MA 535. Algebra**

Fundamentals of group theory: homomorphisms and the isomorphism theorems, finite groups, structure of finitely generated Abelian groups. Structure of rings: homomorphisms, ideals, factor rings and the isomorphism theorems, integral domains, factorization. Field theory: extension fields, finite fields, theory of equations. Selected topics from: Galois theory, Sylow theory, Jordan-Hölder theory, Polya theory, group presentations, basic representation theory and group characters, modules. Applications chosen from mathematical physics, Gröbner bases, symmetry, cryptography, error-correcting codes, number theory.



### **MA 540/4631. Probability and Mathematical Statistics I**

Intended for advanced undergraduates and beginning graduate students in the mathematical sciences, and for others intending to pursue the mathematical study of probability and statistics. Topics covered include axiomatic foundations, the calculus of probability, conditional probability and independence, Bayes' Theorem, random variables, discrete and continuous distributions, joint, marginal and conditional distributions, covariance and correlation, expectation, generating functions, exponential families, transformations of random variables, types of convergence, laws of large numbers the Central Limit Theorem, Taylor series expansion, the delta method. (Prerequisite: knowledge of basic probability at the level of MA 2631 and of advanced calculus at the level of MA 3831/3832 is assumed.)

### **MA 541/4632. Probability and Mathematical Statistics II**

This course is designed to provide background in principles of statistics. Topics covered include estimation criteria: method of moments, maximum likelihood, least squares, Bayes, point and interval estimation, Fisher's information, Cramer-Rao lower bound, sufficiency, unbiasedness, and completeness, Rao-Blackwell Theorem, efficiency, consistency, interval estimation pivotal quantities, Neyman-Person Lemma, uniformly most powerful tests, unbiased, invariant and similar tests, likelihood ratio tests, convex loss functions, risk functions, admissibility and minimaxity, Bayes decision rules. (Prerequisite: knowledge of the material in MA 540 is assumed.)

### **MA 542. Regression Analysis**

Regression analysis is a statistical tool that utilizes the relation between a response variable and one or more predictor variables for the purposes of description, prediction and/or control. Successful use of regression analysis requires an appreciation of both the theory and the practical problems that often arise when the technique is employed with real-world data. Topics covered include the theory and application of the general linear regression model, model fitting, estimation and prediction, hypothesis testing, the analysis of variance and related distribution theory, model diagnostics and remedial measures, model building and validation, and generalizations such as logistic response models and Poisson regression. Additional topics may be covered as time permits. Application of theory to real-world problems will be emphasized using statistical computer packages. (Prerequisite: knowledge of probability and statistics at the level of MA 511 and of matrix algebra is assumed.)

### **MA 543/DS 502. Statistical Methods for Data Science**

Statistical Methods for Data Science surveys the statistical methods most useful in data science applications. Topics covered include predictive modeling methods, including multiple linear regression, and time series, data dimension reduction, discrimination and classification

methods, clustering methods, and committee methods. Students will implement these methods using statistical software. Prerequisites: Statistics at the level of MA 2611 and MA 2612 and linear algebra at the level of MA 2071.

### **MA 546. Design and Analysis of Experiments**

Controlled experiments—studies in which treatments are assigned to observational units—are the gold standard of scientific investigation. The goal of the statistical design and analysis of experiments is to (1) identify the factors which most affect a given process or phenomenon; (2) identify the ways in which these factors affect the process or phenomenon, both individually and in combination; (3) accomplish goals 1 and 2 with minimum cost and maximum efficiency while maintaining the validity of the results. Topics covered in this course include the design, implementation and analysis of completely randomized complete block, nested, split plot, Latin square and repeated measures designs. Emphasis will be on the application of the theory to real data using statistical computer packages. (Prerequisite: knowledge of basic probability and statistics at the level of MA 511 is assumed.)

### **MA 547. Design and Analysis of Observational and Sampling Studies**

Like controlled experiments, observational studies seek to establish cause-effect relationships, but unlike controlled experiments, they lack the ability to assign treatments to observational units. Sampling studies, such as sample surveys, seek to characterize aspects of populations by obtaining and analyzing samples from those populations. Topics from observational studies include: prospective and retrospective studies; overt and hidden bias; adjustments by stratification and matching. Topics from sampling studies include: simple random sampling and associated estimates for means, totals, and proportions; estimates for subpopulations; unequal probability sampling; ratio and regression estimation; stratified, cluster, systematic, multistage, double sampling designs, and, time permitting, topics such as model-based sampling, spatial and adaptive sampling. (Prerequisite: knowledge of basic probability and statistics, at the level of MA 511 is assumed.)

### **MA 548. Quality Control**

This course provides the student with the basic statistical tools needed to evaluate the quality of products and processes. Topics covered include the philosophy and implementation of continuous quality improvement methods, Shewhart control charts for variables and attributes, EWMA and Cusum control charts, process capability analysis, factorial and fractional factorial experiments for process design and improvement, and response surface methods for process optimization. Additional topics will be covered as time permits. Special emphasis will be placed on realistic applications of the theory using statistical computer packages. (Prerequisite: knowledge of basic probability and statistic, at the level of MA 511 is assumed.)

### **MA 549. Analysis of Lifetime Data**

Lifetime data occurs frequently in engineering, where it is known as reliability or failure time data, and in the biomedical sciences, where it is known as survival data. This course covers the basic methods for analyzing such data. Topics include: probability models for lifetime data, censoring, graphical methods of model selection and analysis, parametric and distribution-free inference, parametric and distribution-free regression methods. As time permits, additional topics such as frailty models and accelerated life models will be considered. Special emphasis will be placed on realistic applications of the theory using statistical computer packages. (Prerequisite: knowledge of basic probability and statistics at the level of MA 511 is assumed.)

### **MA 550. Time Series Analysis**

Time series are collections of observations made sequentially in time. Examples of this type of data abound in many fields ranging from finance to engineering. Special techniques are called for in order to analyze and model these data. This course introduces the student to time and frequency domain techniques, including topics such as autocorrelation, spectral analysis, and ARMA and ARIMA models, Box-Jenkins methodology, fitting, forecasting, and seasonal adjustments. Time permitting, additional topics will be chosen from: Kalman filter, smoothing techniques, Holt-Winters procedures, FARIMA and GARCH models, and joint time-frequency methods such as wavelets. The emphasis will be in application to real data situations using statistical computer packages. (Prerequisite: knowledge of MA 511 is assumed. Knowledge of MA 541 is also assumed, but may be taken concurrently.)

### **MA 552. Distribution-Free and Robust Statistical Methods**

Distribution-free statistical methods relax the usual distributional modeling assumptions of classical statistical methods. Robust methods are statistical procedures that are relatively insensitive to departures from typical assumptions, while retaining the expected behavior when assumptions are satisfied. Topics covered include, time permitting, order statistics and ranks; classical distribution-free tests such as the sign, Wilcoxon signed rank, and Wilcoxon rank sum tests, and associated point estimators and confidence intervals; tests pertaining to one and two-way layouts; the Kolmogorov-Smirnov test; permutation methods; bootstrap and Monte Carlo methods; M, L, and R estimators, regression, kernel density estimation and other smoothing methods. Comparisons will be made to standard parametric methods. (Prerequisite: knowledge of MA 541 is assumed, but may be taken concurrently.)

### **MA 554. Applied Multivariate Analysis**

This course is an introduction to statistical methods for analyzing multivariate data. Topics covered are multivariate sampling distributions, tests and estimation of multivariate normal parameters, multivariate ANOVA, regression,

discriminant analysis, cluster analysis, factor analysis and principal components. Additional topics will be covered as time permits. Students will be required to analyze real data using one of the standard packages available. (Prerequisite: knowledge of MA 541 is assumed, but may be taken concurrently. Knowledge of matrix algebra is assumed.)

### **MA 556. Applied Bayesian Statistics**

Bayesian statistics makes use of an inferential process that models data summarizing the results in terms of probability distributions for the model parameters. A key feature is that in the Bayesian approach, past information can be updated with new data in an elegant way in order to aid in decision making. Topics included in the courses: statistical decision theory, the Bayesian inferential framework (model specification, model fitting and model checking); computational methods for posterior simulation integration; regression models, hierarchical models, and ANOVA; time permitting, additional topics will include generalized linear models, multivariate models, missing data problems, and time series analysis. (Prerequisites: knowledge of MA 541 is assumed.)

### **MA 557 Graduate Seminar in Analysis and Applied Mathematics**

(1 credit)

This seminar introduces students to modern issues in Analysis and Applied Mathematics. During the seminar, students and faculty will present and discuss recent research papers from the literature. Students will gain insights about current advances in the mathematical sciences and their applications.

### **MA 559. Statistics Graduate Seminar**

(1 credit)

This seminar introduces students to issues and trends in modern statistics. In the seminar, students and faculty will read and discuss survey and research papers, make and attend presentations, and participate in brainstorming sessions toward the solution of advanced statistical problems.

### **MA 560. Graduate Seminar**

(0 credits)

Designed to introduce graduate students to study of original papers and afford them opportunity to give account of their work by talks in the seminar.

### **MA 562 A and B.**

#### **Professional Master's Seminar**

(0 credits)

This seminar will introduce professional master's students to topics related to general writing, presentation, group communication and interviewing skills, and will provide the foundations to successful cooperation within interdisciplinary team environments. All full-time students will be required to take both components A and B of the seminar during their professional master's studies.

### **MA 571. Financial Mathematics I**

This course provides an introduction to many of the central concepts in mathematical finance. The focus of the course is on arbitrage-based pricing of derivative securities. Topics include stochastic calculus, securities markets, arbitrage-based pricing of options and their uses for hedging and risk management, forward and futures contracts, European options, American options, exotic options, binomial stock price models, the Black-Scholes-Merton partial differential equation, risk-neutral option pricing, the fundamental theorems of asset pricing, sensitivity measures ("Greeks"), and Merton's credit risk model. (Prerequisite: MA 540, which can be taken concurrently.)

### **MA 572. Financial Mathematics II**

The course is devoted to the mathematics of fixed income securities and to the financial instruments and methods used to manage interest rate risk. The first topics covered are the term-structure of interest rates, bonds, futures, interest rate swaps and their uses as investment or hedging tools and in asset-liability management. The second part of the course is devoted to dynamic term-structure models, including risk-neutral interest rate trees, the Heath-Jarrow-Morton model, Libor market models, and forward measures. Applications of these models are also covered, including the pricing of non-linear interest rate derivatives such as caps, floors, collars, swaptions and the dynamic hedging of interest rate risk. The course concludes with the coverage of mortgage-backed and asset-backed securities. (Prerequisite: MA 571.)

### **MA 573. Computational Methods of Financial Mathematics**

Most realistic quantitative finance models are too complex to allow explicit analytic solutions and are solved by numerical computational methods. The first part of the course covers the application of finite difference methods to the partial differential equations and interest rate models arising in finance. Topics included are explicit, implicit and Crank-Nicholson finite difference schemes for fixed and free boundary value problems, their convergence and stability. The second part of the course covers Monte Carlo simulation methods, including random number generation, variance reduction techniques and the use of low discrepancy sequences. (Prerequisites: MA 571 and programming skills at the level of MA 579, which can be taken concurrently.)

### **MA 574. Portfolio Valuation and Risk Management**

Balancing financial risks vs returns by the use of asset diversification is one of the fundamental tasks of quantitative financial management. This course is devoted to the use of mathematical optimization and statistics to allocate assets, to construct and manage portfolios and to measure and manage the resulting risks. The first part of the course covers Markowitz's mean-variance optimization and efficient frontiers, Sharpe's single index and capital asset pricing models, arbitrage pricing theory, structural and statistical multifactor models, risk allocation and risk budgeting. The second part of the course is devoted to

the intertwining of optimization and statistical methodologies in modern portfolio management, including resampled efficiency, robust and Bayesian statistical methods, the Black-Litterman model and robust portfolio optimization.

### **MA 575. Market and Credit Risk Models and Management**

The objective of the course is to familiarize students with the most important quantitative models and methods used to measure and manage financial risk, with special emphasis on market and credit risk. The course starts with the introduction of metrics of risk such as volatility, value-at-risk and expected shortfall and with the fundamental quantitative techniques used in financial risk evaluation and management. The next section is devoted to market risk including volatility modeling, time series, non-normal heavy tailed phenomena and multivariate notions of dependence such as copulas, correlations and tail-dependence. The final section concentrates on credit risk including structural and dynamic models and default contagion and applies the mathematical tools to the valuation of default contingent claims including credit default swaps, structured credit portfolios and collateralized debt obligations. (Prerequisite: knowledge of MA 540 assumed but can be taken concurrently.)

### **MA 579. Financial Programming Workshop**

(1 or 2 credits)

The objective is to elevate the students' computer programming skills to the semi-professional level required in quantitative finance. Participants learn through hands-on experience by working on a structured set of mini projects from computational finance under the guidance of an experienced trainer and the faculty in charge. The programming language used may be C++, MATLAB, R/S, VB or another language widely used in quantitative finance and may alternate from year to year. (Prerequisite: Intermediate scientific programming skills.)

### **MA 584/BCB 504. Statistical Methods in Genetics and Bioinformatics**

This course provides students with knowledge and understanding of the applications of statistics in modern genetics and bioinformatics. The course generally covers population genetics, genetic epidemiology, and statistical models in bioinformatics. Specific topics include meiosis modeling, stochastic models for recombination, linkage and association studies (parametric vs. nonparametric models, family-based vs. population-based models) for mapping genes of qualitative and quantitative traits, gene expression data analysis, DNA and protein sequence analysis, and molecular evolution. Statistical approaches include log-likelihood ratio tests, score tests, generalized linear models, EM algorithm, Markov chain Monte Carlo, hidden Markov model, and classification and regression trees. Students may not receive credit for both MA 584 and MA 4603. (Prerequisite: knowledge of probability and statistics at the undergraduate level.)

### **MA 590. Special Topics**

Courses on special topics are offered under this number. Contact the Mathematical Sciences Department for current offerings.

### **MA 595. Independent Study**

(1 to 3 credits)

Supervised independent study of a topic of mutual interest to the instructor and the student.

### **MA 596. Master's Capstone**

(1 or more credits)

The Master's Capstone is designed to integrate classroom learning with real-world practice. It can consist of a project, a practicum, a research review report or a research proposal. A written report and a presentation are required.

### **MA 598. Professional Master's Project**

(1 or more credits)

This project will provide the opportunity to apply and extend the material studied in the coursework to the study of a real-world problem originating in the industry. The project will be a capstone integrating industrial experience with the previously acquired academic knowledge and skills. The topic of the project will come from a problem generated in industry, and could originate from prior internship or industry experience of the student. The student will prepare a written project report and make a presentation before a committee including the faculty advisor, at least one additional WPI faculty member and representatives of a possible industrial sponsor. The advisor of record must be a faculty member of the WPI Mathematical Sciences Department. The student must submit a written project proposal for approval by the Graduate Committee prior to registering for the project.

### **MA 599. Thesis**

(1 or more credits)

Research study at the master's level.

### **MA 698. Ph.D. Project**

(1 or more credits)

Ph.D. project work.

### **MA 699. Dissertation**

(1 or more credits)

Research study at the Ph.D. level.

## **Mathematics for Educators**

### **MME 518. Geometrical Concepts**

This course focuses primarily on the foundations and applications of Euclidean and non-Euclidean geometries. The rich and diverse nature of the subject also implies the need to explore other topics, for example, chaos and fractals. The course incorporates collaborative learning and the investigation of ideas through group projects. Possible topics include geometrical software and computer graphics, tiling and tessellations, two- and three-dimensional geometry, inversive geometry, graphical representations of functions, model construction, fundamental relationship between algebra and geometry, applications of geometry, geometry transformations and projective geometry, and convexity.

### **MME 522. Applications of Calculus**

(2 credits)

There are three major goals for this course: to establish the underlying principles of calculus, to reinforce students' calculus skills through investigation of applications involving those skills, and to give students the opportunity to develop projects and laboratory assignments for use by first-year calculus students. The course will focus heavily on the use of technology to solve problems involving applications of calculus concepts. In addition, MME students will be expected to master the mathematical rigor of these calculus concepts so that they will be better prepared to develop their own projects and laboratory assignments. For example, if an MME student chose to develop a lab on convergence of sequence, he/she would be expected to understand the rigorous definition of convergence and how to apply it to gain sufficient and/or necessary conditions for convergence. The process of developing these first-year calculus assignments will enable the MME students to increase their own mathematical understanding of concepts while learning to handle mathematical and computer issues which will be encountered by their own calculus students. Their understanding of the concepts and applications of calculus will be further reinforced through computer laboratory assignments and group projects. Applications might include exponential decay of drugs in the body, optimal crankshaft design, population growth, or development of cruise control systems. (Prerequisite: MME 532)

### **MME 523. Analysis with Applications**

(2 credits)

This course introduces students to mathematical analysis and its use in modeling. It will emphasize topics of calculus (including multidimensional) in a rigorous way. These topics will be motivated by their usefulness for understanding concepts of the calculus and for facilitating the solutions of engineering and science problems. Projects involving applications and appropriate use of technology will be an essential part of the course. Topics covered may include dynamical systems and differential equations; growth and decay; equilibrium; probabilistic dynamics; optimal decisions and reward; applying, building and validating models; functions on  $n$ -vectors; properties of functions; parametric equations; series; applications such as pendulum problems; electromagnetism; vibrations; electronics; transportation; gravitational fields; and heat loss. (Prerequisite: MME 532)

### **MME/SEME 524-25. Probability, Statistics and Data Analysis I, II**

(4 credits)

This course introduces students to probability, the mathematical description of random phenomena, and to statistics, the science of data. Students in this course will acquire the following knowledge and skills:

- Probability models-mathematical models used to describe and predict random phenomena. Students will learn several basic probability models and their uses, and will obtain experience in modeling random phenomena.
- Data analysis-the art/science of finding patterns in data and using those patterns to explain the process which produced the data. Students will be able to explore and draw conclusions about data using computational and graphical methods. The iterative nature of statistical exploration will be emphasized.
- Statistical inference and modeling-the use of data sampled from a process and the probability model of that process to draw conclusions about the process. Students will attain proficiency in selecting, fitting and criticizing models, and in drawing inference from data.
- Design of experiments and sampling studies – the proper way to design experiments and sampling studies so that statistically valid inferences can be drawn. Special attention will be given to the role of experiments and sampling studies in scientific investigation. Through lab and project work, students will obtain practical skills in designing and analyzing studies and experiments. Course topics will be motivated whenever possible by applications and reinforced by experimental and computer lab experiences. One in-depth project per semester involving design, data collection, and statistical or probabilistic analysis will serve to integrate and consolidate student skills and understanding. Students will be expected to learn and use a statistical computer package such as MINITAB.

### **MME 526-27. Linear Models I, II**

(4 credits)

This two-course sequence imparts computational skills, particularly those involving matrices, to deepen understanding of mathematical structure and methods of proof; it also includes discussion on a variety of applications of the material developed, including linear optimization. Topics in this sequence may include systems of linear equations, vector spaces, linear independence, bases, linear transformations, determinants, eigenvalues and eigenvectors, systems of linear inequalities, linear programming problems, basic solutions, duality and game theory. Applications may include economic models, computer graphics, least squares approximation, systems of differential equations, graphs and networks, and Markov processes. (Prerequisite: MME 532)

### **MME 528. Mathematical Modeling and Problem Solving**

(2 credits)

This course introduces students to the process of developing mathematical models as a means for solving real problems. The course will encompass several different modeling situations that utilize a variety of mathematical topics. The mathematical fundamentals of these topics will be discussed, but with continued reference to their use in finding



the solutions to problems. Problems to be covered include balance in small group behavior, traffic flow, air pollution flow, group decision making, transportation, assignment, project planning and the critical path method, genetics, inventory control and queueing. (Prerequisite: MME 532)

### **MME 529. Numbers, Polynomials and Algebraic Structures**

(2 credits)

This course enables secondary mathematics teachers to see how commonly taught topics such as number systems and polynomials fit into the broader context of algebra. The course will begin with treatment of arithmetic, working through Euclid's algorithm and its applications, the fundamental theorem of arithmetic and its applications, multiplicative functions, the Chinese remainder theorem and the arithmetic of  $\mathbb{Z}/n$ . This information will be carried over to polynomials in one variable over the rational and real numbers, culminating in the construction of root fields for polynomials via quotients of polynomial rings. Arithmetic in the Gaussian integers and the integers in various other quadratic fields (especially the field of cube roots of unity) will be explored through applications such as the generation of Pythagorean triples and solutions to other Diophantine equations (like finding integer-sided triangles with a 60 degree angle). The course will then explore cyclotomy, and the arithmetic in rings of cyclotomic integers. This will culminate in Gauss's construction of the regular 5-gon and 17-gon and the impossibility of constructing a 9-gon or trisecting a 60-degree angle. Finally, solutions of cubics and quartics by radicals will be studied. All topics will be based on the analysis of explicit calculations with (generalized) numbers. The proposed curriculum covers topics that are part of the folklore for high school mathematics (the impossibility of certain ruler and compass constructions), but that many teachers know only as facts. There are also many applications of the ideas that will allow the teachers to use results and ideas from abstract algebra to construct for their students problems that have manageable solutions.

### **MME 531. Discrete Mathematics**

(3 credits)

This course deals with concepts and methods which emphasize the discrete nature in many problems and structures. The rapid growth of this branch of mathematics has been inspired by its wide range of applicability to diverse fields such as computer science, management, and biology. The essential ingredients of the course are:

**Combinatorics -The Art of Counting.** Topics include basic counting principles and methods such as recurrence relations, generating functions, the inclusion-exclusion principle and the pigeon-hole principle. Applications may include block designs, latin squares, finite projective planes, coding theory, optimization and algorithmic analysis.

**Graph Theory.** This includes direct graphs and networks. Among the parameters to be examined are traversibility, connectivity, planarity, duality and colorability.

### **MME 532. Differential Equations**

(2 credits)

This course would have concepts and techniques for both Ordinary and Partial Differential Equations. Topics from ordinary differential equations include existence and uniqueness for first order, single variable problems as well as separation of variables and linear methods for first order problems. Second order, linear equations would be solved for both the homogeneous and non homogeneous cases. The phenomena of beats and resonance would be analyzed. The Laplace Transform would be introduced for appropriate second order nonhomogeneous problems. Partial Differential Equations would focus on boundary value problems arising from the Heat and Wave equations in one variable. Fourier Series expansions would be used to satisfy initial conditions and the concepts of orthogonality and convergence addressed.

### **MME 592/SEME 602. Project Preparation (Part of a 3-course sequence with MME 594 and MME 596)**

(2 credits) (ISG)

Students will research and develop a mathematical topic or pedagogical technique. The project will typically lead to classroom implementation; however, a project involving mathematical research at an appropriate level of rigor will also be acceptable. Preparation will be completed in conjunction with at least one faculty member from the Mathematical Sciences Department and will include exhaustive research on the proposed topic. The course will result in a detailed proposal that will be presented to the MME Project Committee for approval; continuation with the project is contingent upon this approval.

### **MME 594/SEME 604. Project Implementation**

(2 credits) (ISG)

Students will implement and carry out the project developed during the project preparation course. Periodic contact and/or observations will be made by the project advisor (see MME 592 Project Preparation) in order to provide feedback and to ensure completion of the proposed task. Data for the purpose of evaluation will be collected by the students throughout the term, when appropriate. If the project includes classroom implementation, the experiment will last for the duration of a semester.

### **MME 596/SEME 606. Project Analysis and Report**

(2 credits) (ISG)

Students will complete a detailed statistical analysis of any data collected during the project implementation using techniques from MME 524-525 Probability, Statistics, and Data Analysis. The final report will be a comprehensive review of the relevant literature, project description, project implementation, any statistical results and conclusions. Project reports will be subject to approval by the MME Project committee and all students will be required to present their project to the mathematical sciences faculty. Course completion is contingent upon approval of the report and satisfactory completion of the presentation.